Fast Marching and Geodesic Methods. Some Applications

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Some joint works with F. Benmansour, Y. Rouchdy, J. Mille, G. Peyré, H. Li , A. Yezzi, Da Chen and J.M. Mirebeau.

Huawei, February 3rd, 2017

01/02/2017 19:16 Laurent D. COHEN, Huawei 2017 1

Overview

- Minimal Paths, Fast Marching and Front Propagation
- Anisotropic Minimal Paths and Tubular model
- Finding contours as a set of minimal paths
- Application to 2D and 3D tree structures
- Geodesic Density for tree structures

01/02/2017 19:16 Laurent D. COHEN, Huawei 2017 26

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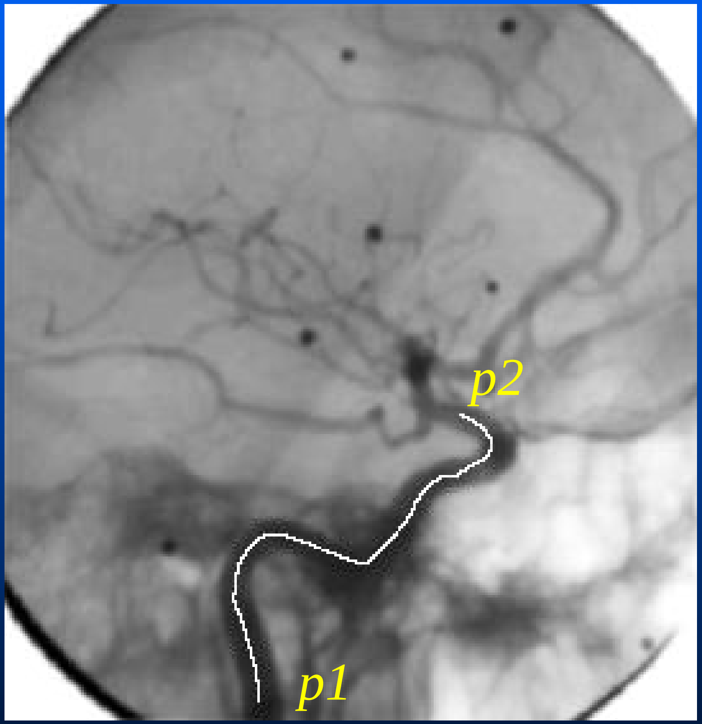
Paths of minimal energy



Looking for a path along which a feature Potential P(x,y) is minimal

$$E(C)=\int_0^L P(C(s))ds$$

example: a vessel
dark structure
P =gray level

Input : Start point $p1=(x1,y1)$

End point $p2=(x2,y2)$

Image

Output: Minimal Path

01/02/2017 19:16 Laurent D. COHEN, Huawei 2017 30

Minimal Paths: Eikonal Equation

$$E(C) = \int_0^L P(C(s))ds$$

Potential P>0 takes lower values near interesting features : on contours, dark structures, ...

STEP 1 : search for the surface of minimal action *U* of *p1* as the minimal energy integrated along a path between start point *p1* and any point *p* in the image

$$\text{Start point } C(0) = p1;$$

$$U_{p1}(p) = \inf_{C(0)=p1;C(L)=p} E(C) = \inf_{C(0)=p1;C(L)=p} \int_0^L P(C(s))ds$$

STEP 2: Back-propagation from the end point *p2* to the start point *p1*:

Simple Gradient Descent along U_{p1}

01/02/2017 19:16 Laurent D. COHEN, Huawei 2017 32

Minimal Paths: Eikonal Equation

STEP 1 : minimal action *U* of *p1* as the minimal energy integrated along a path between start point *p1* and any point *p* in the image

$$\text{Start point } C(0)=p1;$$
$$U_{p1}(p)=\inf_{C(0)=p1;C(L)=p} E(C)=\inf_{C(0)=p1;C(L)=p}\int_0^L P(C(s))ds$$

$$\left\|\nabla U_{p1}(x)\right\| = P(x) \text{ and } U_{p1}(p1)=0$$

Example P=1, U Euclidean distance to p1
in general, U weighted geodesic distance to p1

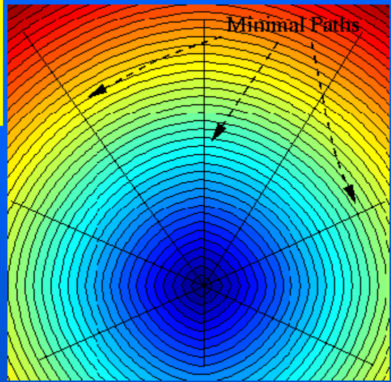
01/02/2017 19:16 Laurent D. COHEN, Huawei 2017 33

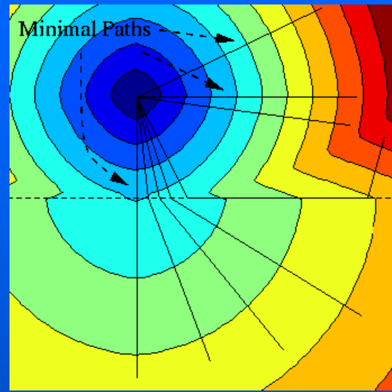
Minimal paths - 2D simple examples

$$E(C)=\int_0^L \tilde{P}(C(s))ds$$

$$U_{p1}(p)=\inf_{C(0)=p1;C(L)=p} E(C)$$

P=c





P1

slower

P2 < P1

faster

Examples of shortest paths on univalued or bivalued potential

Fermat Principle in Geometric Optics :
Path followed by light minimizes time

$$T=\frac{1}{c}\int_{p1}^{p2} n(C(s))\ ds$$

where n>1 is refraction index v=c/n

Snell-Descartes law

01/02/2017 19:16 Laurent D. COHEN, Huawei 2017 34

Minimal Paths and Front Propagation

Minimal Action $U_{p0}(p) = \inf_{C(0)=p0;C(L)=p} \int_0^L \tilde{P}(C(s))\,ds$

Front Propagation $\mathcal{L}(t) = \left\{ p \in R^2 / U_{p0}(p) = t \right\}$

Evolution of t level set of U from p0

$$\frac{\partial \mathcal{L}(\sigma,t)}{\partial t} = \frac{1}{P(\mathcal{L}(\sigma,t))} \vec{n}(\sigma,t)$$

n normal vector to a level set of U is in the direction of the Gradient of U, implies **Eikonal Equation** :

$$\left\| \nabla U_{p0}(x) \right\| = P(x) \text{ and } U_{p0}(p0) = 0$$

01/02/2017 19:16 Laurent D. COHEN, Huawei 2017 40

FAST MARCHING in 2D:

very efficient algorithm O(NlogN) for Eikonal Equation

Introduced by Sethian / Tsistsiklis

Numerical approximation of U(xij) as the solution to the discretized problem with upwind finite difference scheme

$$\| \nabla U \| = \tilde{P} \qquad \left(\frac{\partial U}{\partial x}\right)^2 + \left(\frac{\partial U}{\partial y}\right)^2 = \tilde{P}^2$$

$$\max\left(u - U(x_{i-1,j}), u - U(x_{i+1,j}), 0\right)^2 + \max\left(u - U(x_{i,j-1}), u - U(x_{i,j+1}), 0\right)^2 = h^2 \tilde{P}(x_{i,j})^2$$

This 2nd order equation induces that :

action U at {i,j} depends only of the neighbors that have lower actions.

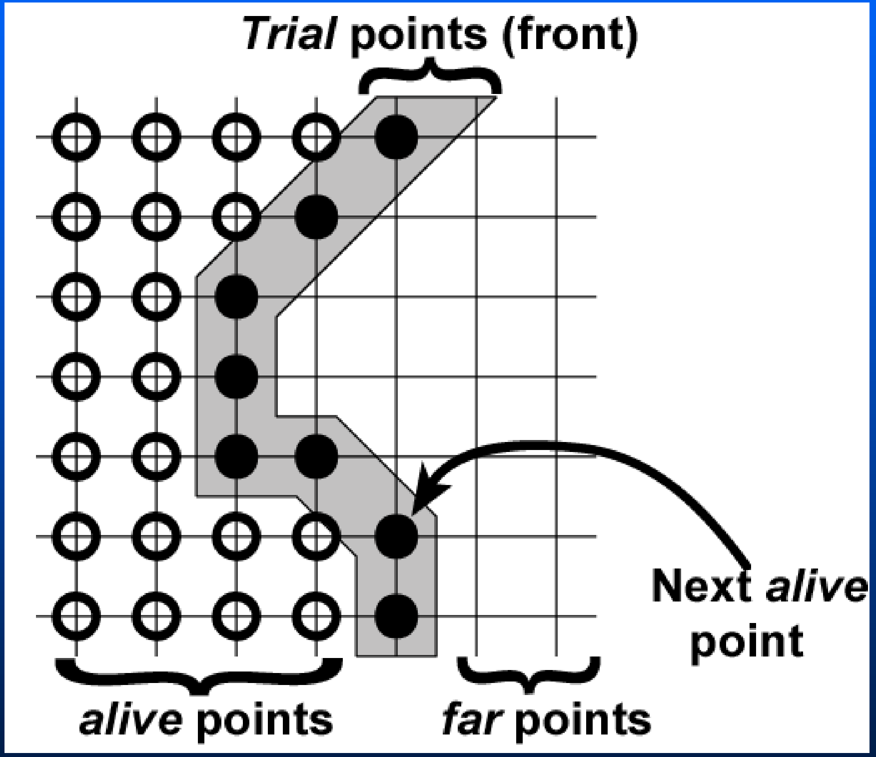
Fast marching introduces order in the selection of the grid points for solving this numerical scheme.

Starting from the initial point p1 with U = 0,

the action computed at each point visited can only grow.

Level sets of U can be seen as a Front propagation outwards.

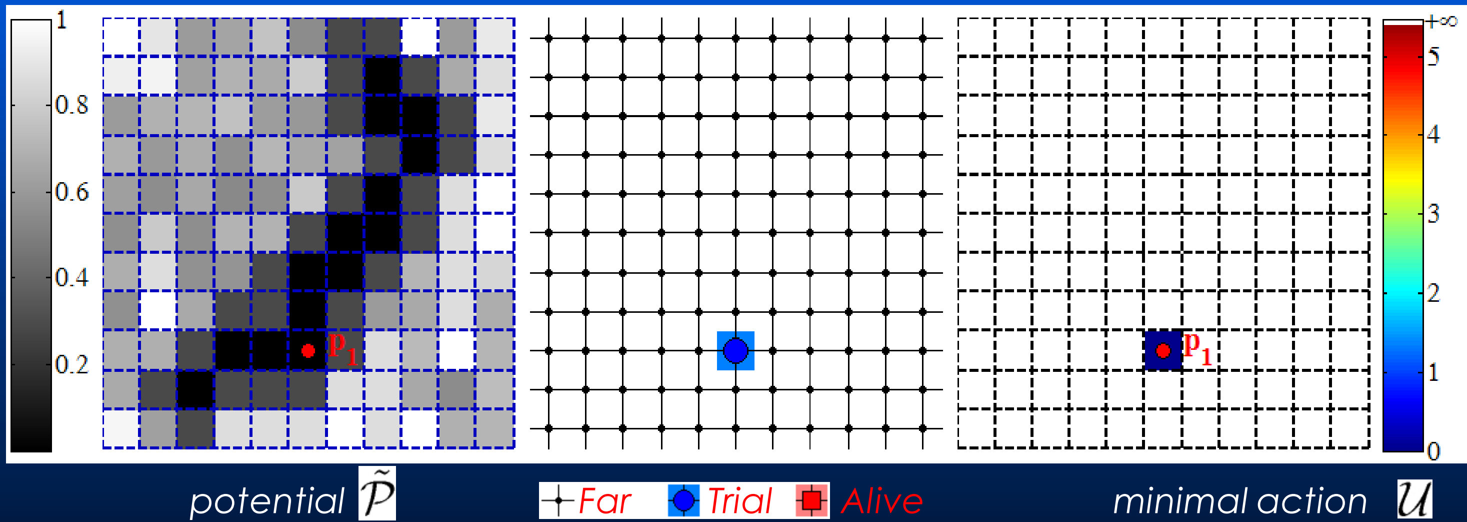
Fast Marching Algorithm (Sethian)



- *Start*: only $\boldsymbol{p_0}$ is *trial* **with *U=0*.**
- *Loop*: ***p*** trial point with minimum ***U*** becomes *alive*. neighbors of ***p*** become *trial* and are updated.

Fast Marching Algorithm

Initialization



J. A. Sethian

A fast marching level set method for monotonically advancing fronts.

P.N.A.S., **93:**1591-1595, 1996.

46

Fast Marching Algorithm

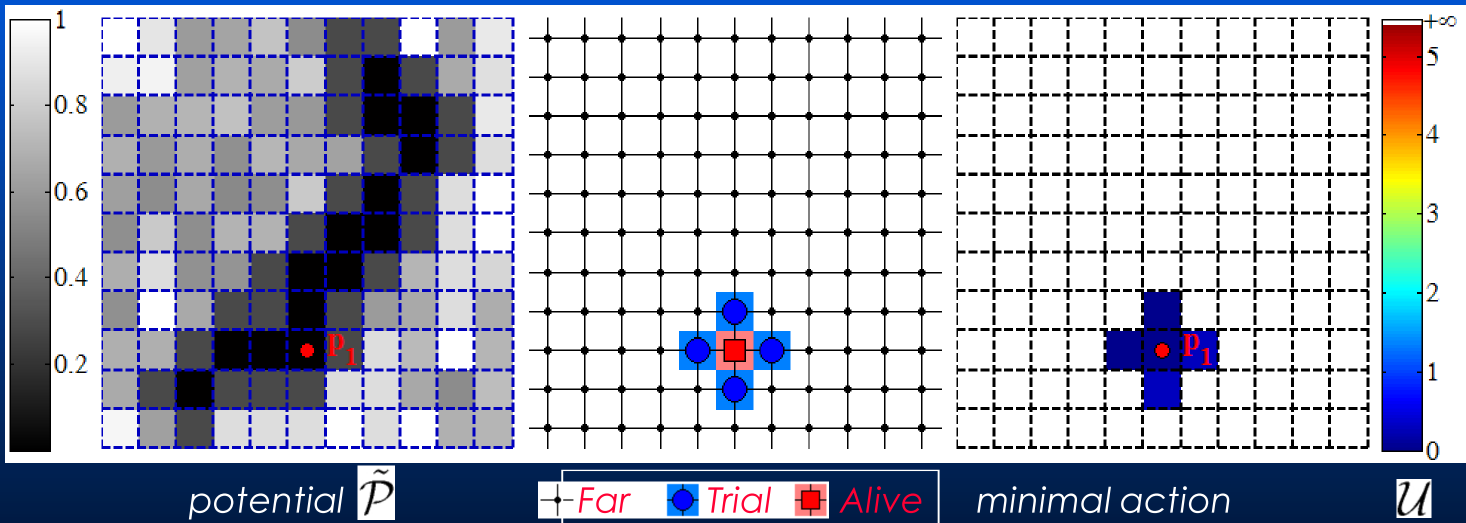
Itération #1

- Find point $\mathbf{x}_{min}$ (*Trial* point with smallest value of $\mathcal{U}$).
- $\mathbf{x}_{min}$ becomes *Alive*.
- For each of 4 neighbors $\mathbf{x}$ of point $\mathbf{x}_{min}$:

  If $\mathbf{x}$ is not *Alive*,

  Estimate $\mathcal{U}(\mathbf{x})$ with upwind scheme.

  $\mathbf{x}$ becomes *Trial*.



J. A. Sethian
A fast marching level set method for monotonically advancing fronts.
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Fast Marching Algorithm

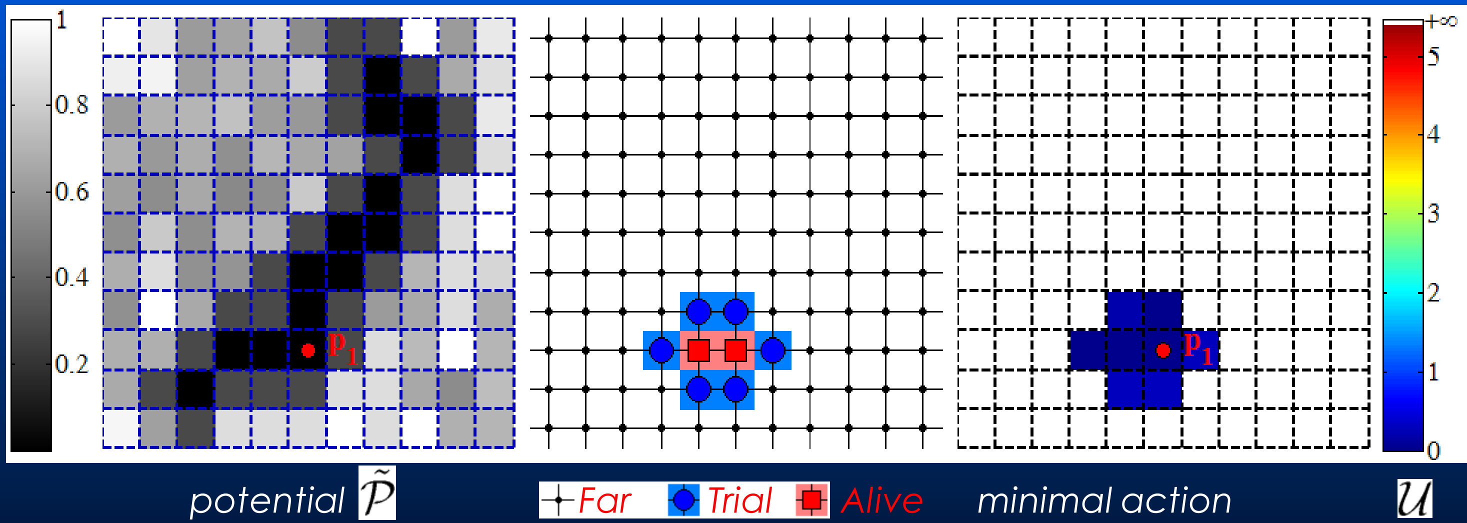
Itération #2

- Find point $\mathbf{x}_{min}$ (*Trial* point with smallest value of $\mathcal{U}$).
- $\mathbf{x}_{min}$ becomes *Alive*.
- For each of 4 neighbors $\mathbf{x}$ of point $\mathbf{x}_{min}$:

  If $\mathbf{x}$ is not *Alive*,

  Estimate $\mathcal{U}(\mathbf{x})$ with upwind scheme.

  $\mathbf{x}$ becomes *Trial*.



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Fast Marching Algorithm

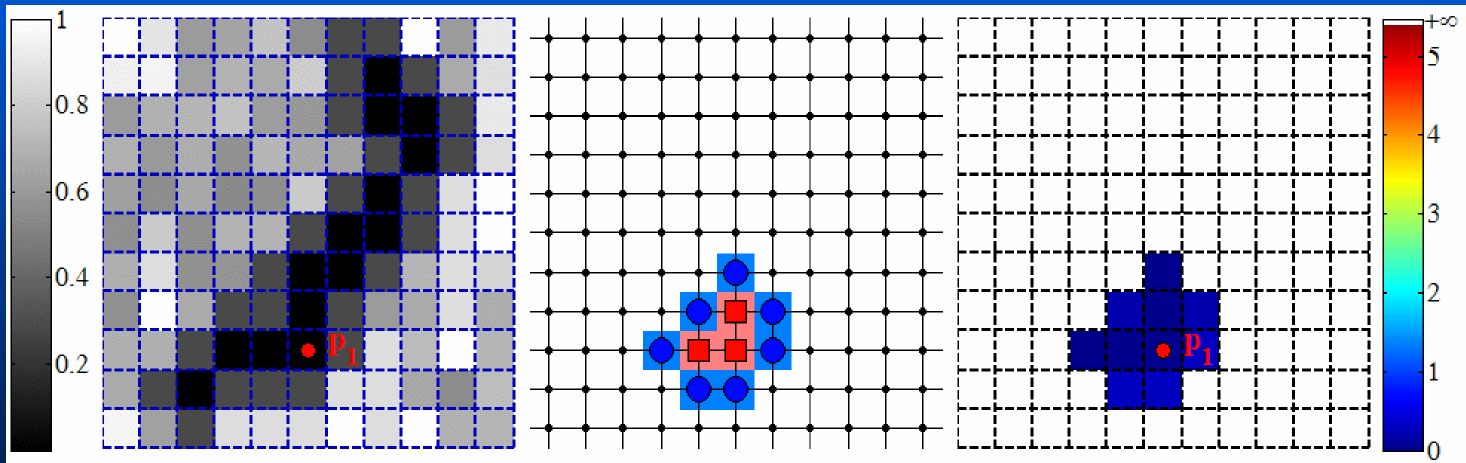
Itération #k

- Find point $\mathbf{x}_{min}$ (*Trial* point with smallest value of $\mathcal{U}$).
- $\mathbf{x}_{min}$ becomes *Alive*.
- For each of 4 neighbors $\mathbf{x}$ of point $\mathbf{x}_{min}$:

  If $\mathbf{x}$ is not *Alive*,

  Estimate $\mathcal{U}(\mathbf{x})$ with upwind scheme.

  $\mathbf{x}$ becomes *Trial*.

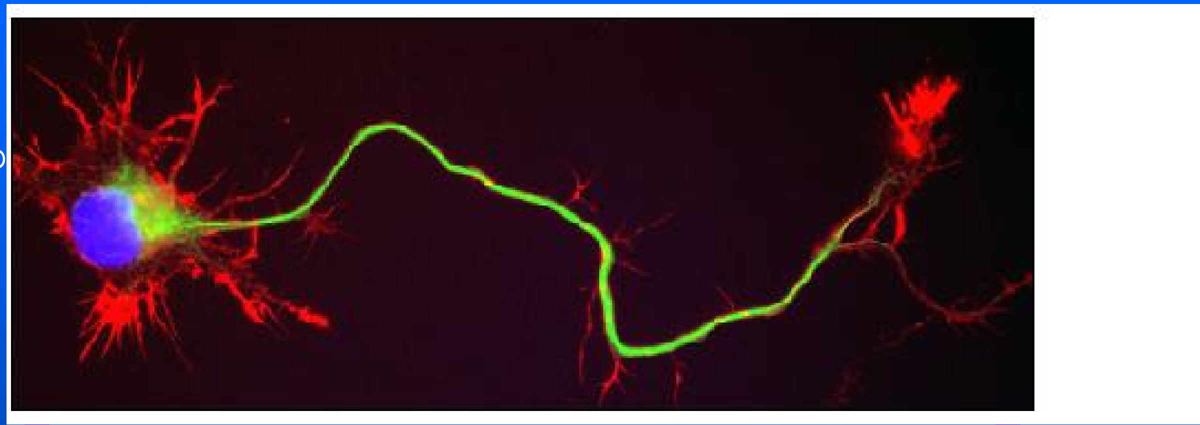


potential $\tilde{\mathcal{P}}$ *Far* *Trial* *Alive* *minimal action* $\mathcal{U}$

J. A. Sethian
A fast marching level set method for monotonically advancing fronts.
P.N.A.S., **93:**1591-1595, 1996.

Minimal Path between p1 and p2

© C. Bouzigues



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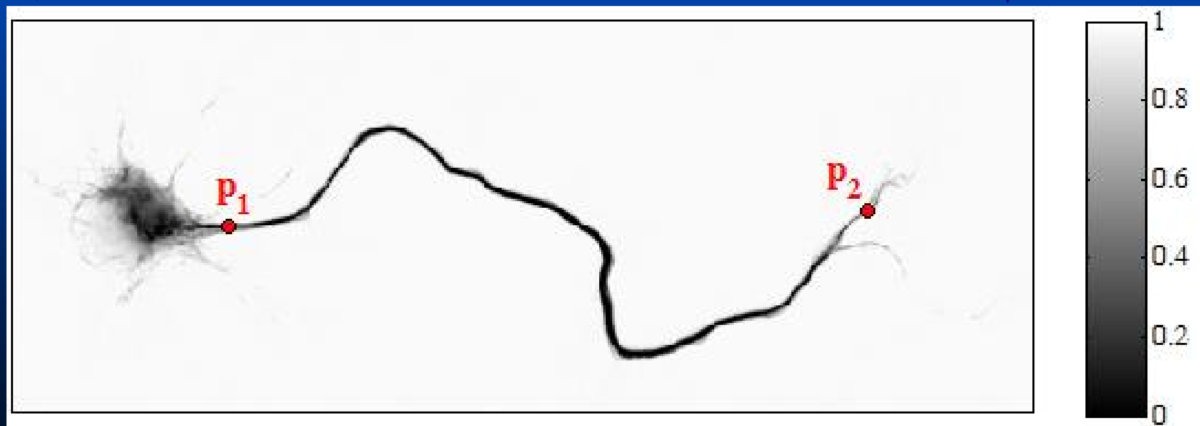


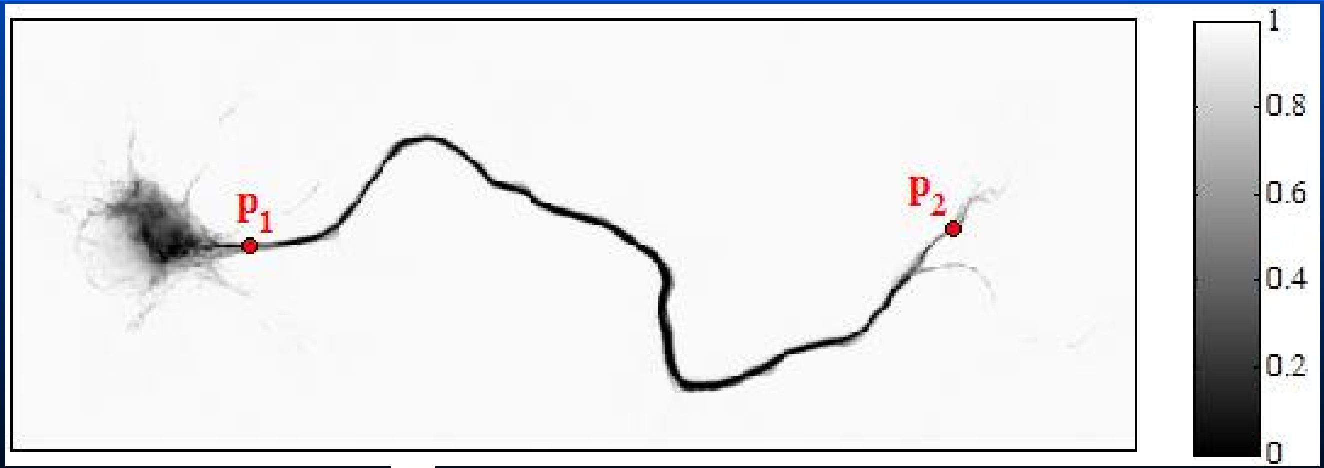
potential $\mathcal{P} : \Omega \to \mathbb{R}^{+*}$

01/02/2017 19:16 Laurent D. COHEN, Huawei 2017 49

Minimal Path between p1 and p2

Step #1

$$\begin{cases} \|\nabla \mathcal{U}_1(\mathbf{x})\| = \tilde{\mathcal{P}}(\mathbf{x}) & \text{pour } \mathbf{x} \in \Omega \\ \mathcal{U}_1(\mathbf{p}_1) = 0 \end{cases}$$

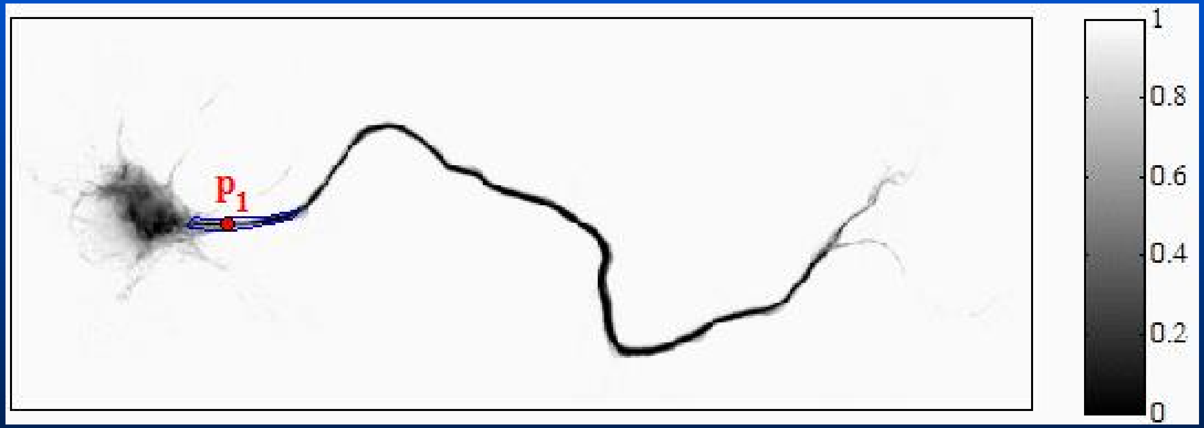


potential $\mathcal{P}$

Minimal Path between p1 and p2

Step #1: U obtained by the FAST MARCHING ALGORITHM

$$\begin{cases} \|\nabla \mathcal{U}_1(\mathbf{x})\| = \tilde{\mathcal{P}}(\mathbf{x}) \ \text{pour } \mathbf{x} \in \Omega \\ \mathcal{U}_1(\mathbf{p}_1) = 0 \end{cases}$$



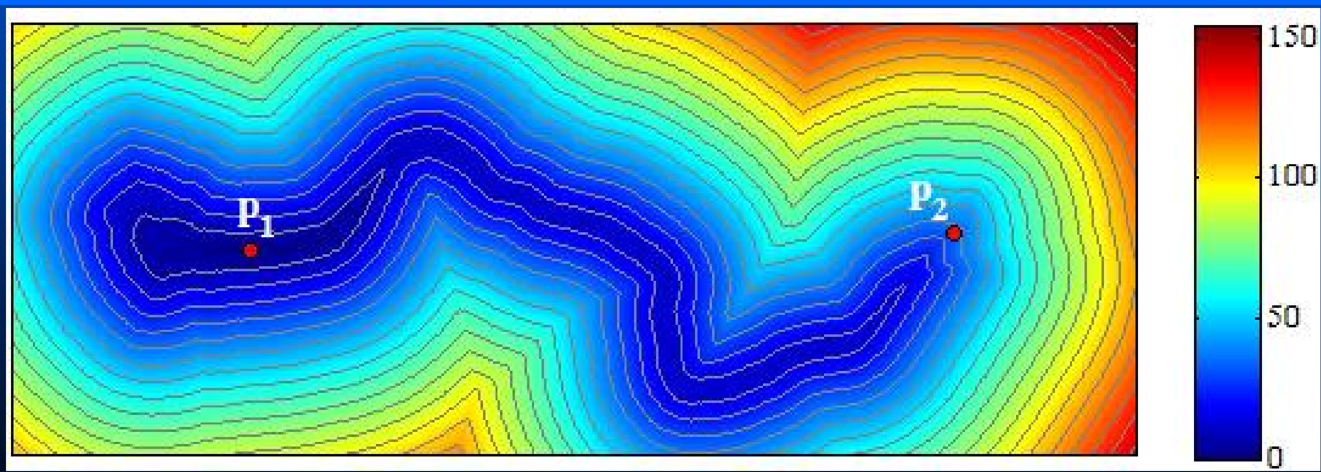
potentiel $\mathcal{P}$

L. D. Cohen, R. Kimmel
Global minimum for active contour models : a minimal path approach.
International Journal of Computer Vision, **25**:57-78, 1997.

Minimal Path between p1 and p2

Step #1: U obtained by the FAST MARCHING ALGORITHM

$$\begin{cases} \|\nabla \mathcal{U}_1(\mathbf{x})\| = \tilde{\mathcal{P}}(\mathbf{x}) \text{ pour } \mathbf{x} \in \Omega \\ \mathcal{U}_1(\mathbf{p}_1) = 0 \end{cases}$$



 minimal action $\mathcal{U}_1$

Minimal Path between p1 and p2

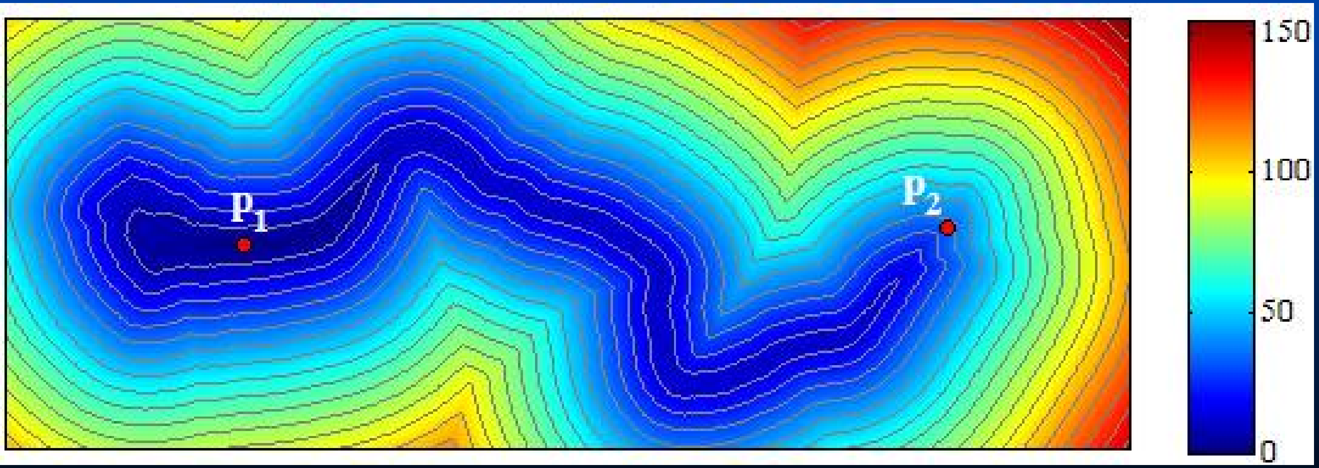
Step #1

$$\begin{cases} \|\nabla \mathcal{U}_1(\mathbf{x})\| = \tilde{\mathcal{P}}(\mathbf{x}) \ \text{ pour } \mathbf{x} \in \Omega \\ \mathcal{U}_1(\mathbf{p}_1) = 0 \end{cases}$$

Step #2

gradient descent on $\mathcal{U}_1$ for extraction of minimal path $\mathcal{C}_{\mathbf{p}_1,\mathbf{p}_2}$

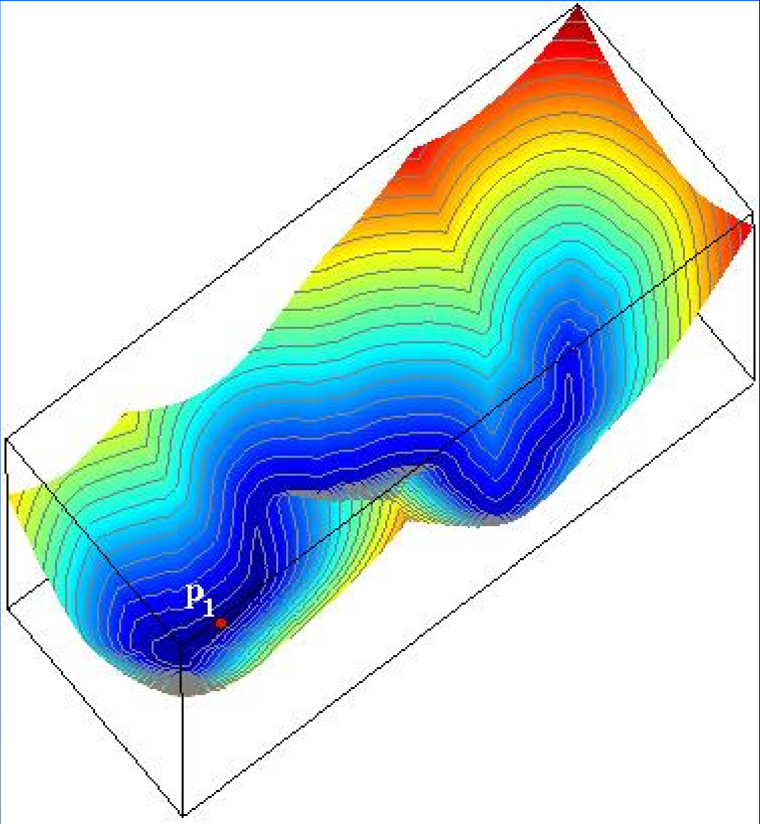
$$\begin{cases} \dfrac{\partial \mathcal{C}_{\mathbf{p}_1,\mathbf{p}_2}(s)}{\partial s} = -\nabla \mathcal{U}_1\left(\mathcal{C}_{\mathbf{p}_1,\mathbf{p}_2}(s)\right) \\ \mathcal{C}_{\mathbf{p}_1,\mathbf{p}_2}(0) = \mathbf{p}_2 \end{cases}$$



minimal action $\mathcal{U}_1$

01/02/2017 19:16 Laurent D. COHEN, Huawei 2017 54

Minimal Path between p1 and p2



minimal action $\mathcal{U}_1$

L. D. Cohen, R. Kimmel
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International Journal of Computer Vision, **25**:57-78, 1997.

Minimal Path between p1 and p2

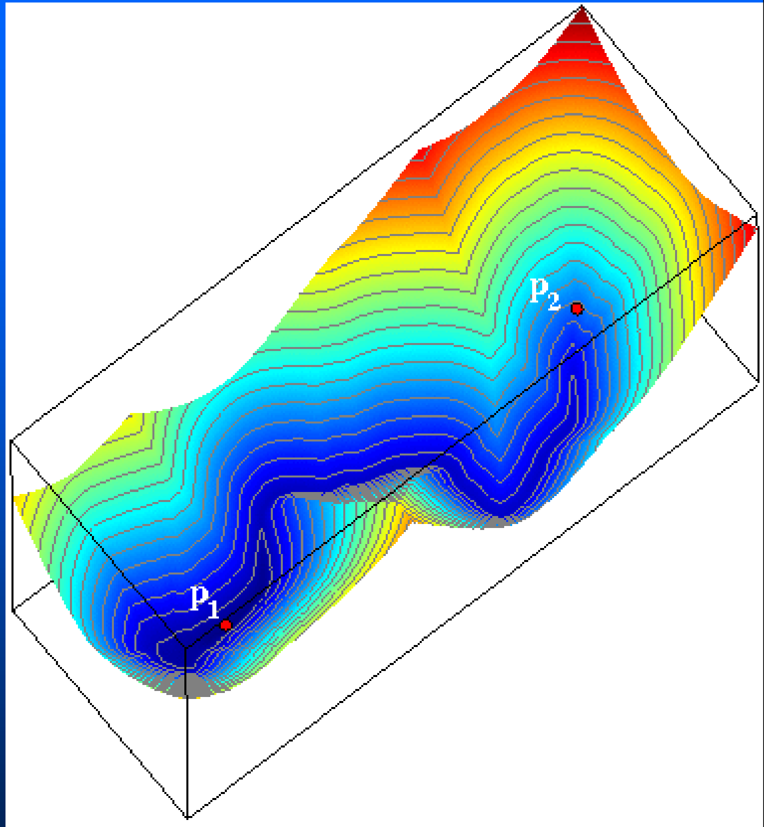
minimal path

$$\mathcal{C}_{\mathbf{p}_1,\mathbf{p}_2} = \min_{\gamma \in \mathcal{A}_{\mathbf{p}_1,\mathbf{p}_2}} \int_\gamma \tilde{\mathcal{P}}(\gamma(s))\mathrm{d}s$$

Is obtained by solving ODE:

$$\begin{cases} \dfrac{\partial \mathcal{C}_{\mathbf{p}_1,\mathbf{p}_2}(s)}{\partial s} = -\nabla \mathcal{U}_1\left(\mathcal{C}_{\mathbf{p}_1,\mathbf{p}_2}(s)\right) \\ \mathcal{C}_{\mathbf{p}_1,\mathbf{p}_2}(0) = \mathbf{p}_2 \end{cases}$$

⇨ simple gradient descent on $\mathcal{U}_1$ from $\mathbf{p_2}$ to $\mathbf{p_1}$



minimal action $\mathcal{U}_1$

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Minimal Path between p1 and p2

Step #1

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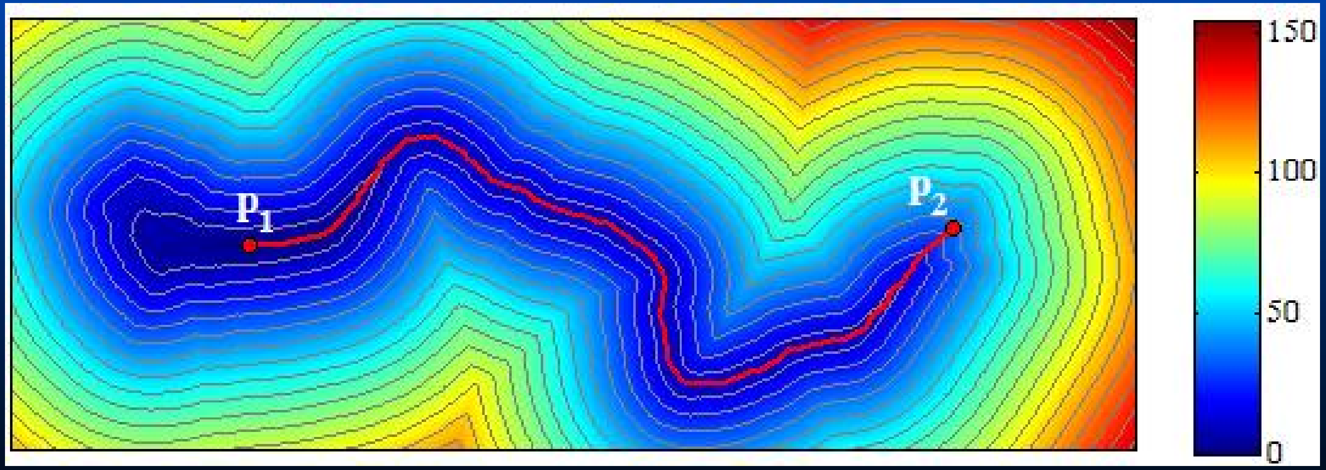
Step #2

gradient descent on $\mathcal{U}_1$ for extraction of minimal path $\mathcal{C}_{\mathbf{p}_1,\mathbf{p}_2}$

$$\begin{cases} \dfrac{\partial \mathcal{C}_{\mathbf{p}_1,\mathbf{p}_2}(s)}{\partial s} = -\nabla \mathcal{U}_1(\mathcal{C}_{\mathbf{p}_1,\mathbf{p}_2}(s)) \\ \mathcal{C}_{\mathbf{p}_1,\mathbf{p}_2}(0) = \mathbf{p}_2 \end{cases}$$



minimal action $\mathcal{U}_1$

01/02/2017 19:16 Laurent D. COHEN, Huawei 2017 57

Minimal Path between p1 and p2

Step #1

$$\begin{cases} \|\nabla \mathcal{U}_1(\mathbf{x})\| = \tilde{\mathcal{P}}(\mathbf{x}) & \text{pour } \mathbf{x} \in \Omega \\ \mathcal{U}_1(\mathbf{p}_1) = 0 \end{cases}$$

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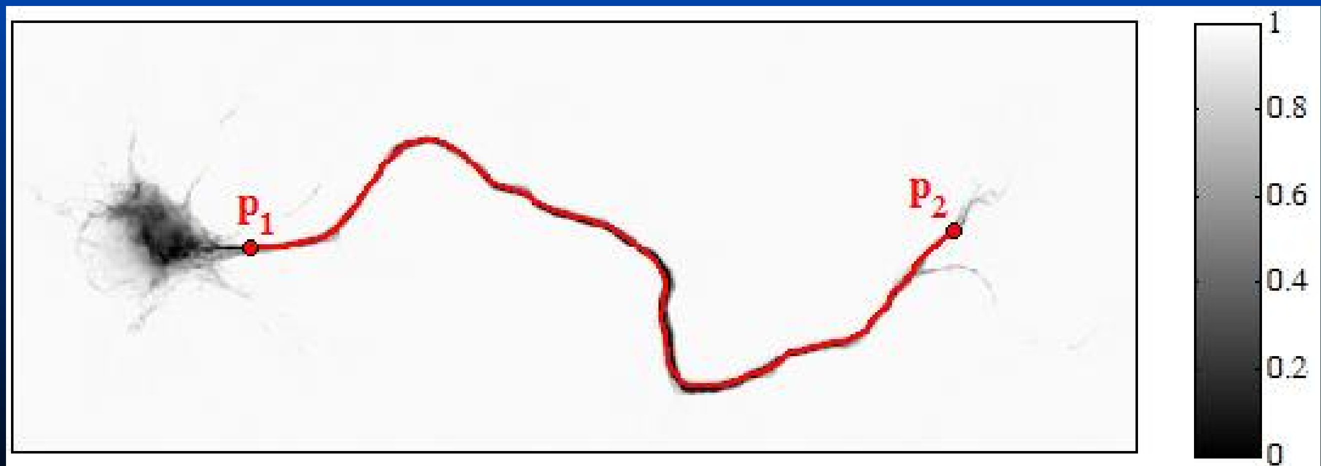
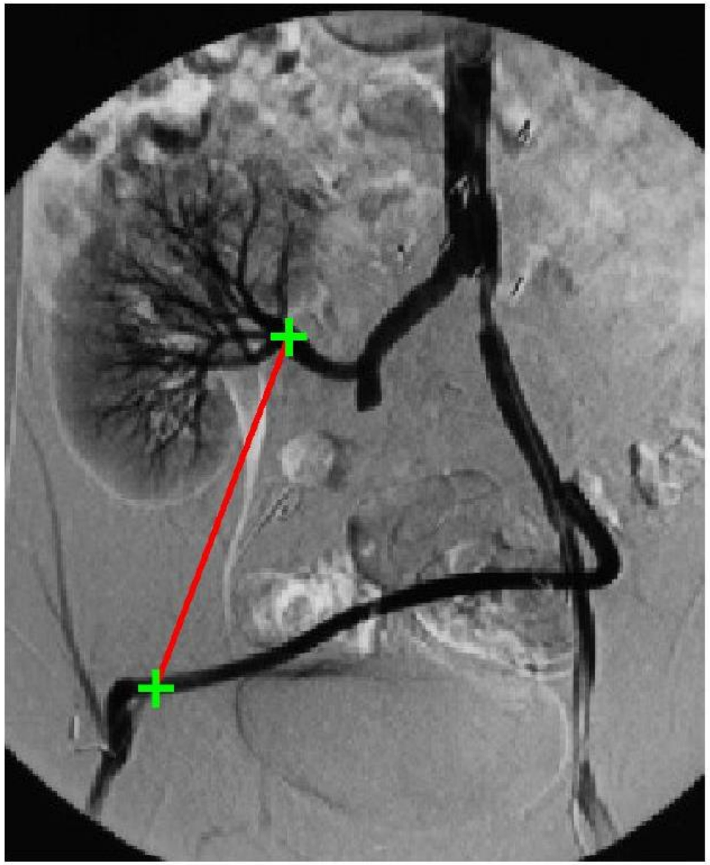


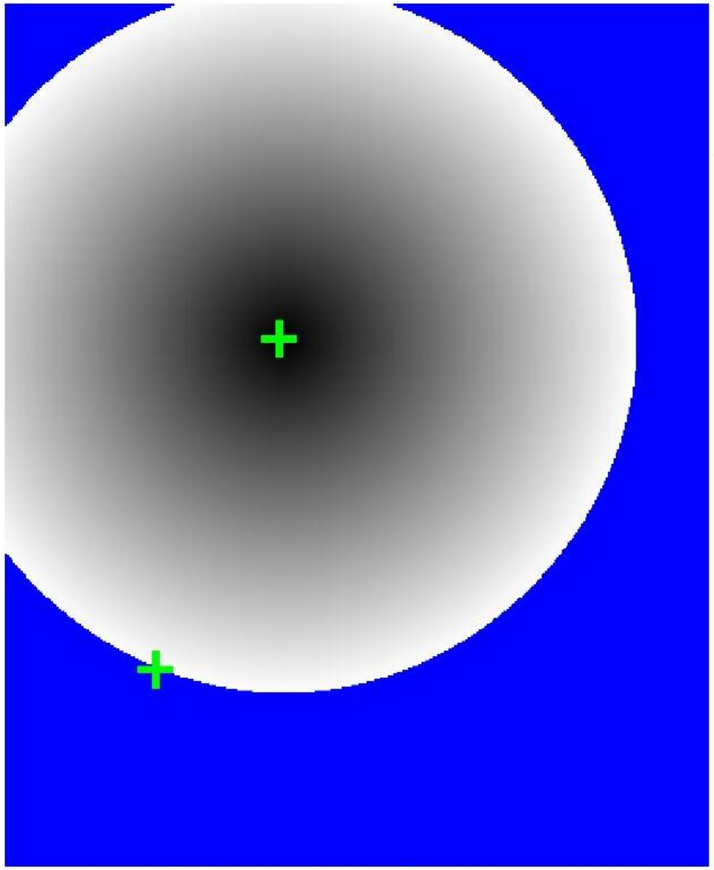
01/02/2017 19:16 *potential* $\mathcal{P}$ Laurent D. COHEN, Huawei 2017 58

Minimal paths for 2D segmentation

- $P(\mathbf{x}) = 1 \Longrightarrow$ droite (plus court chemin euclidien)



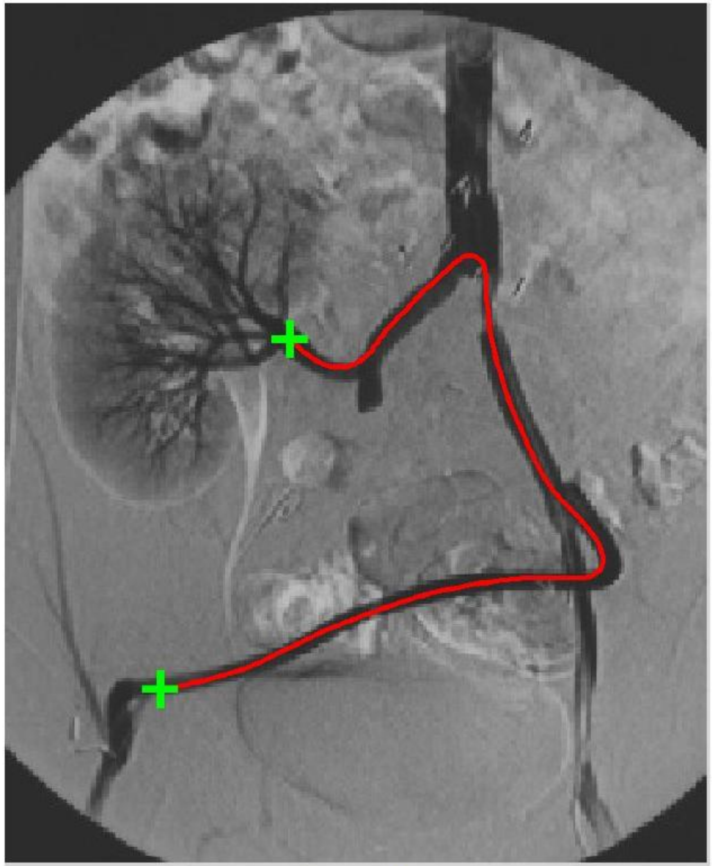
Chemin



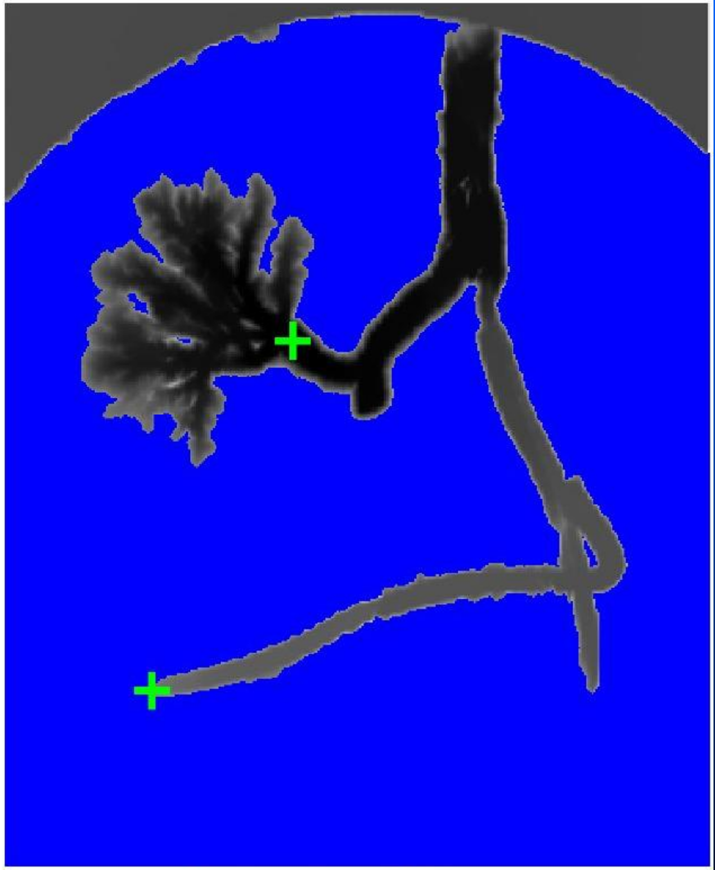
Carte de distance

Minimal paths for 2D segmentation

- $P(\mathbf{x}) = w + (I(\mathbf{x}) - I(\mathbf{x}_0))^2 \Longrightarrow$ chemin d'intensité homogène

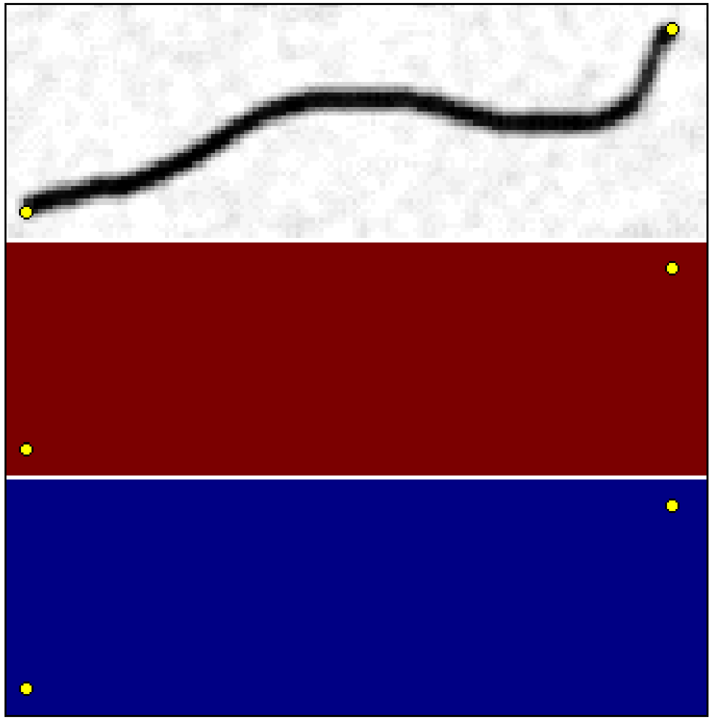


Chemin



Carte de distance

Simultaneous propagation from both ends



Reference:
T. Deschamps and L. D. Cohen
Minimal paths in 3D images and application to virtual endoscopy.
Proceedings ECCV'00, Dublin, Ireland, 2000.

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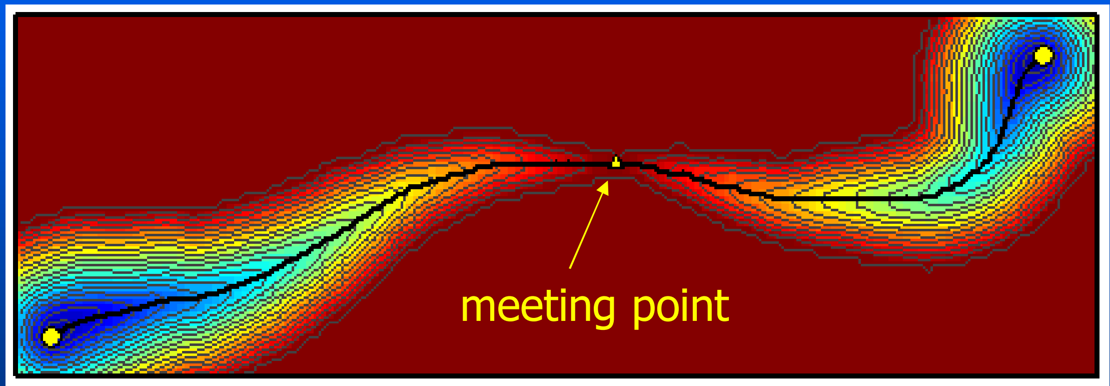
Minimal Paths

Simultaneous propagation of two fronts until a shock occurs.



Reference:
T. Deschamps and L. D. Cohen
Minimal paths in 3D images and application to virtual endoscopy.
Proceedings ECCV'00, Dublin, Ireland, 2000.

Link with Dynamic Programming

- Metrication error -

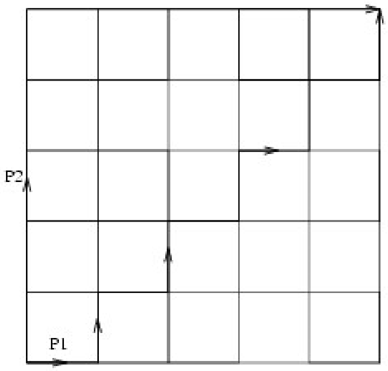


FIG. 22: An L^1 norm cause the shortest path to suffer from errors of up to 41%. In this case both P_1 and P_2 are optimal, and will stay optimal no matter how much we refine the (4-neighboring) grid.

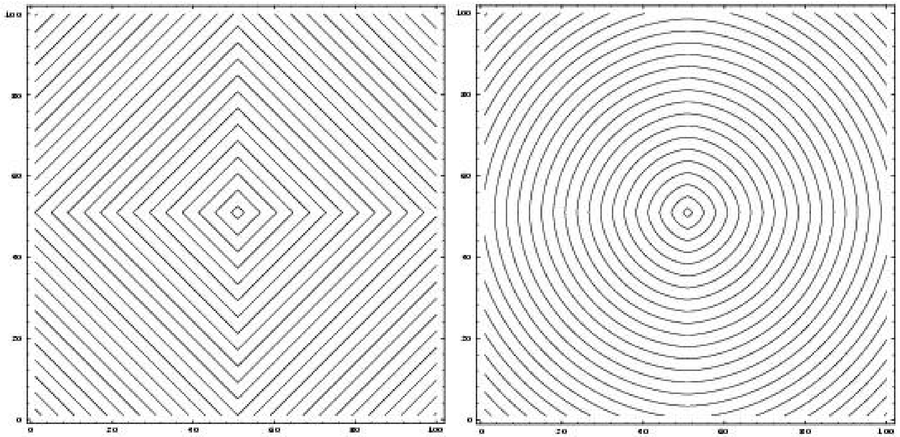


FIG. 23: Illustration of metrication error for computation of the distance map to a single point, showing level sets of the distance. On the left : a graph search-like discrete distance computation gives squares ; On the right : the distance is obtained by our approach, giving circles.

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3D Minimal Path for tubular shapes in 2D Centerline+width

2D in space , 1D for radius of vessel (Li, Yezzi 2007)

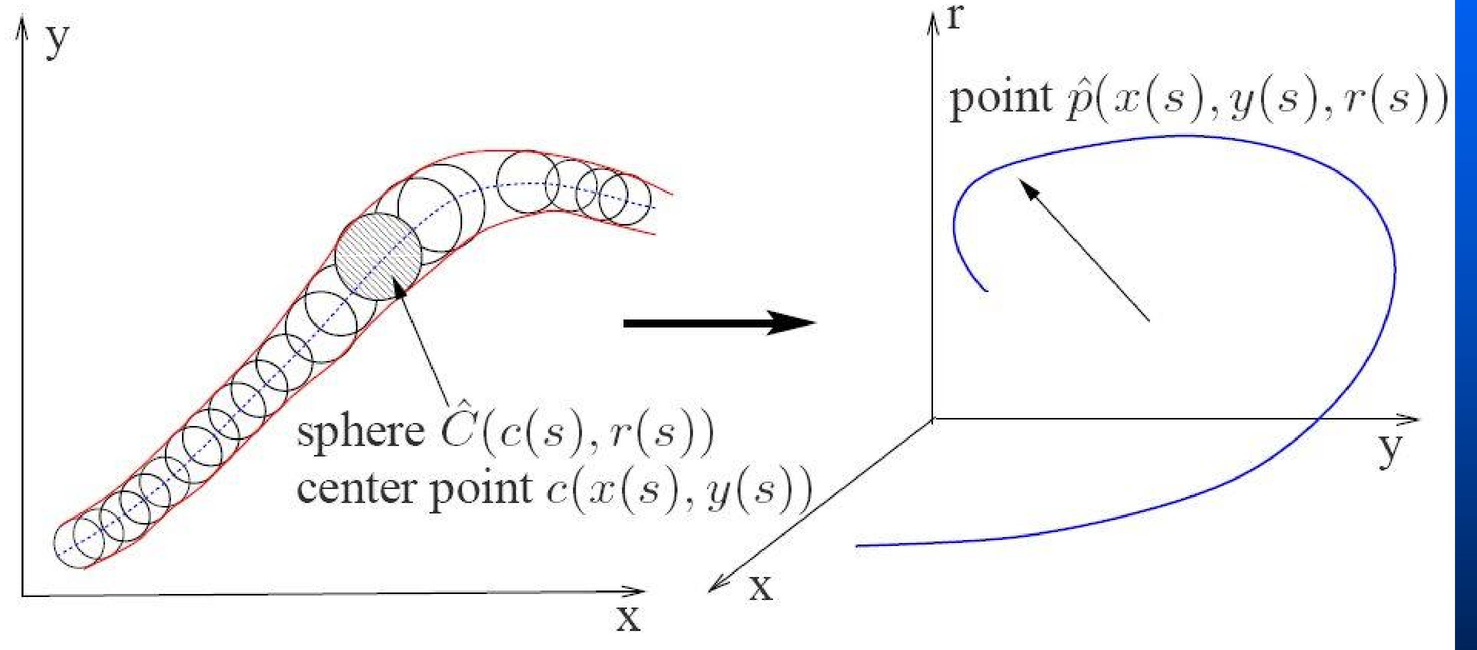


Figure 1. A tubular surface is presented as the envelope of a family of spheres with continuously changing center points and radii.

3D Minimal Path for tubular shapes in 2D

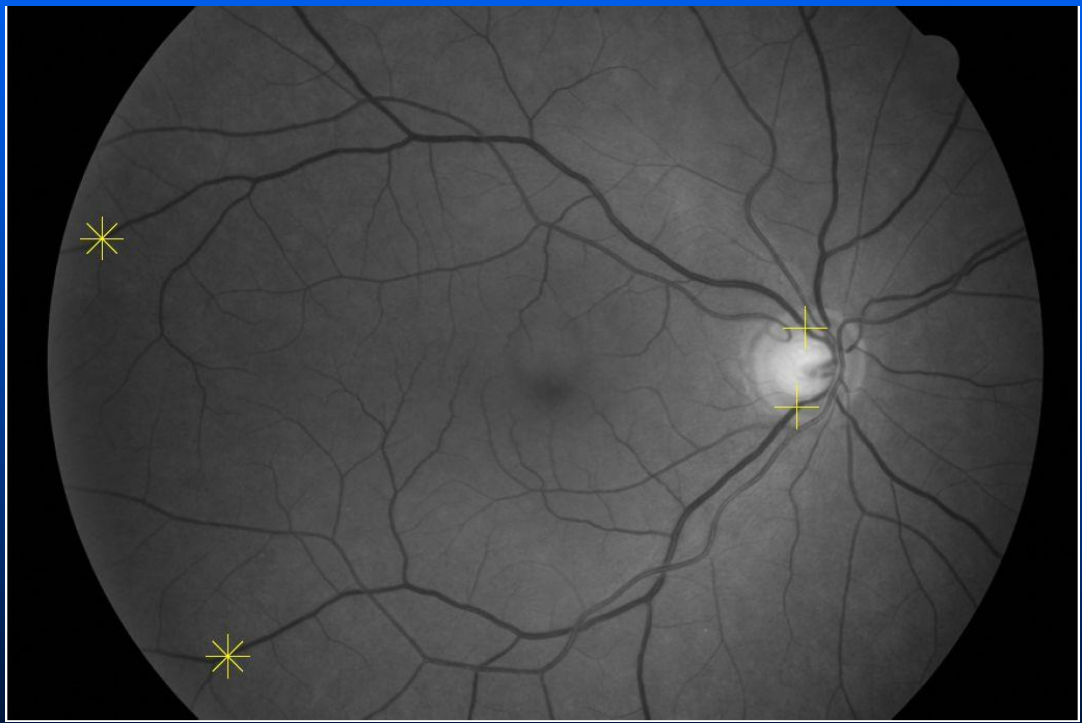
2D in space , 1D for radius of vessel

Fig. 2. Vessel segmentation for an angiogram 2D projection image based on the proposed method

Typical Retina Image

01/02/2017 19:16 Laurent D. COHEN, Huawei 2017 74

Two pairs of user given points



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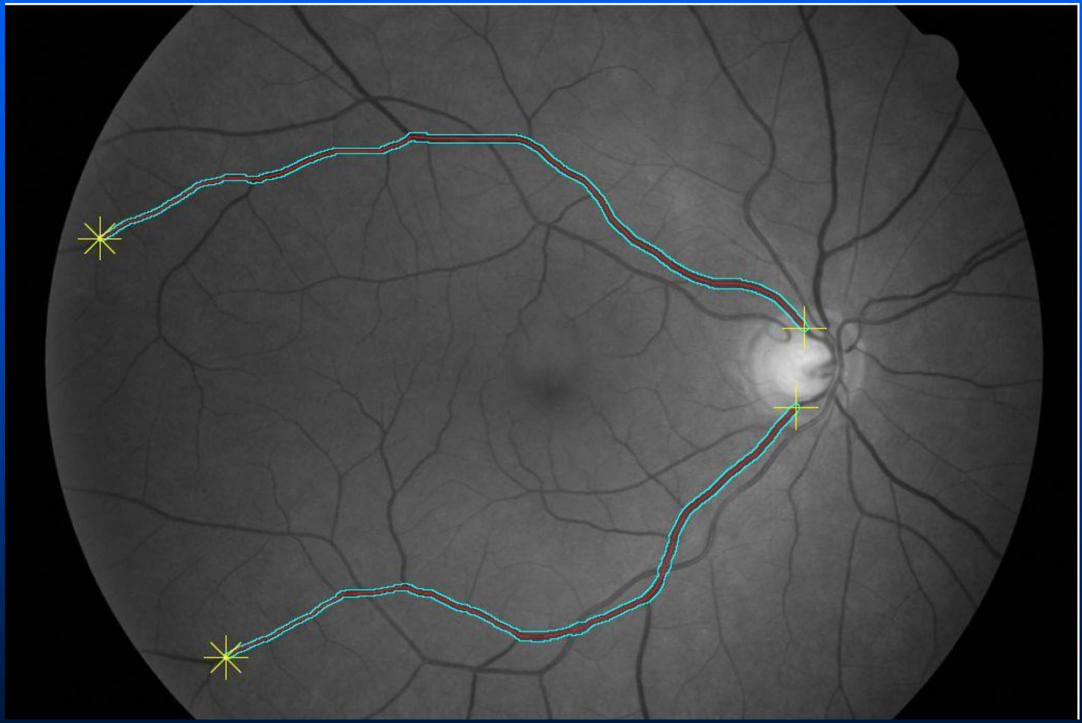
Extraction by 2D+radius minimal path model



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Anisotropic Energy

$$E(C)=\int_0^L P(C(s),C'(s))ds$$

Considers the local orientations of the structures

$$P(C(s),C'(s))=\sqrt{C'(s)^T H(C(s))C'(s)}$$

Describes an infinitesimal distance along an oriented pathway *C*, relative to a metric H

Geodesic Methods for Shape and Surface Processing, Gabriel Peyre and Laurent D. Cohen in Advances in Computational Vision and Medical Image Processing: Methods and Applications, Springer, 2009.

Anisotropic Energy: Eikonal Equation

$$E(C) = \int_0^L \sqrt{C'(s)^T H(C(s)) C'(s)}\, ds$$

Start point $C(0) = p1$; $U_{p1}(p) = \inf_{C(0)=p1; C(L)=p} E(C)$

$$\left\| \nabla U_{p1}(p) \right\|_{H(p)^{-1}} = \sqrt{\nabla U_{p1}{}^T H^{-1} \nabla U_{p1}} = 1$$

and $U_{p1}(p1) = 0$

Geodesic Methods for Shape and Surface Processing, Gabriel Peyre and Laurent D. Cohen in Advances in Computational Vision and Medical Image Processing: Methods and Applications, Springer, 2009.

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Anisotropic Energy: Gradient descent

$$E(C)=\int_0^L \sqrt{C'(s)^T H(C(s))C'(s)}\,ds$$

$\text{Start point } C(0)=p1;$ $\quad U_{p1}(p)=\inf_{C(0)=p1;C(L)=p} E(C)$

$$C'(s)=-H^{-1}(C(s))\nabla U_{p1}(C(s))$$

$$\text{and } U_{p1}(p1)=0$$

Geodesic Methods for Shape and Surface Processing, Gabriel Peyre and Laurent D. Cohen in Advances in Computational Vision and Medical Image Processing: Methods and Applications, Springer, 2009.

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Anisotropic Energy: includes Isotropic case

$$E(C)=\int_0^L \sqrt{C'(s)^T H(C(s))C'(s)}ds$$

Start point $C(0)=p1;$

$$H(p)=P^2(p)Id$$

$$\left\|\nabla U_{p1}(p)\right\|=P$$

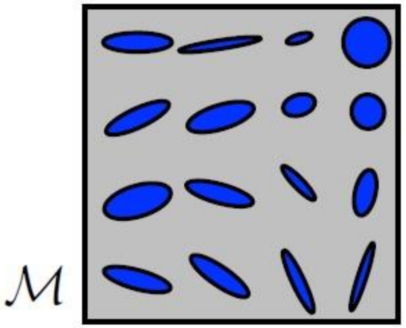
$$C'(t)=-\nabla U_{p1}(C(t))$$

Geodesic Methods for Shape and Surface Processing, Gabriel Peyre and Laurent D. Cohen in Advances in Computational Vision and Medical Image Processing: Methods and Applications, Springer, 2009.

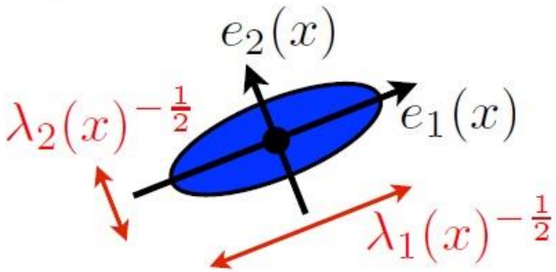
Anisotropy and Geodesics

Tensor eigen-decomposition:

$$H(x) = \lambda_1(x) e_1(x) e_1(x)^{\mathrm{T}} + \lambda_2(x) e_2(x) e_2(x)^{\mathrm{T}} \quad \text{with} \quad 0 < \lambda_1 \leqslant \lambda_2,$$



$$\{\eta \setminus \eta^* H(x) \eta \leqslant 1\}$$

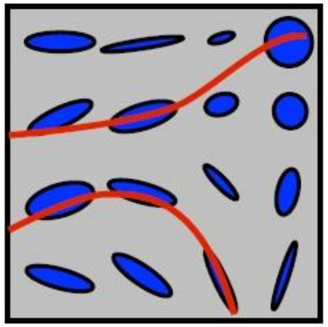


01/02/2017 19:16 Laurent D. COHEN, Huawei 2017 99

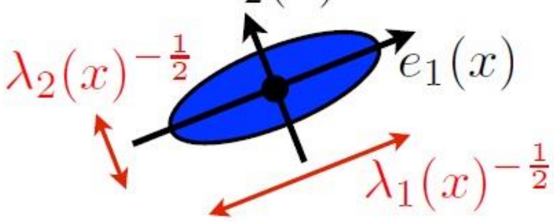
Anisotropy and Geodesics

Tensor eigen-decomposition:

$$H(x) = \lambda_1(x)e_1(x)e_1(x)^{\mathrm{T}} + \lambda_2(x)e_2(x)e_2(x)^{\mathrm{T}} \quad \text{with} \quad 0 < \lambda_1 \leqslant \lambda_2,$$

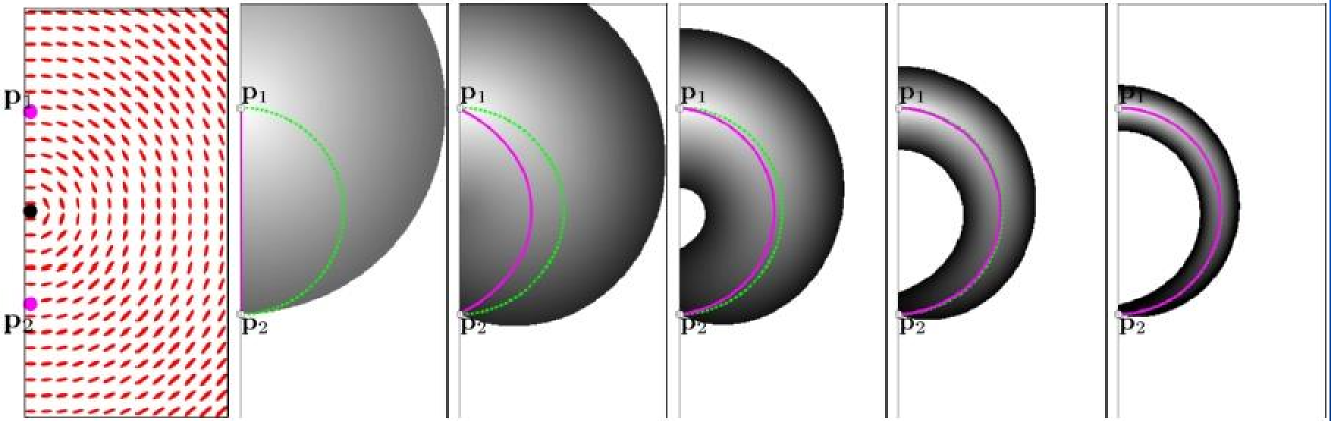
$\mathcal{M}$ 

$\{\eta \setminus \eta^* H(x)\eta \leqslant 1\}$



Geodesics  tend to follow $e_1(x)$.

Anisotropy and Geodesics



Metric $\mathcal{M}$ | $w_2 = w_1$ | $w_2 = 2w_1$ | $w_2 = 4w_1$ | $w_2 = 8w_1$ | $w_2 = 16w_1$

FIG. 2.14: Given an elliptic metric $\mathcal{M} = w_1^2 \mathrm{e}_r \mathrm{e}_r^T + w_2^2 \mathrm{e}_\theta \mathrm{e}_\theta^T$ with standard polar notations, influence of anisotropy ratio $\frac{w_2}{w_1}$ is shown.

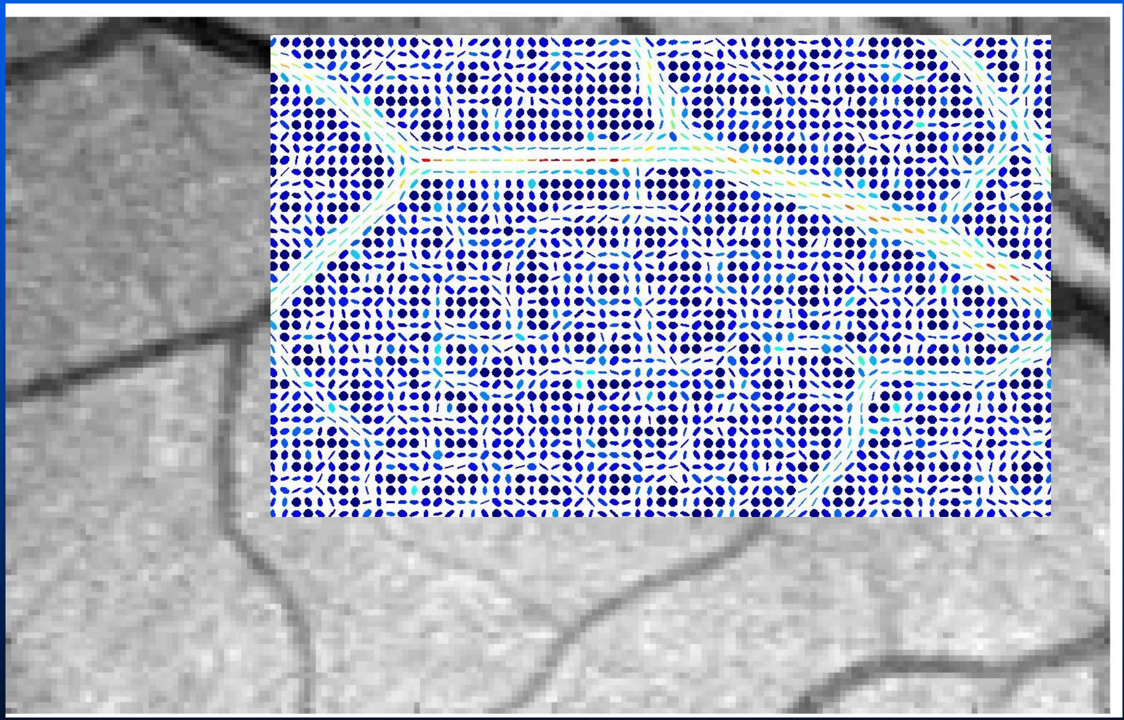
01/02/2017 19:16 Laurent D. COHEN, Huawei 2017 102

Orientation-Dependent Energy

(with Benmansour, CVPR'09, IJCV'10)

$$E(C) = \int_0^L P(C(s), C'(s))\, ds$$

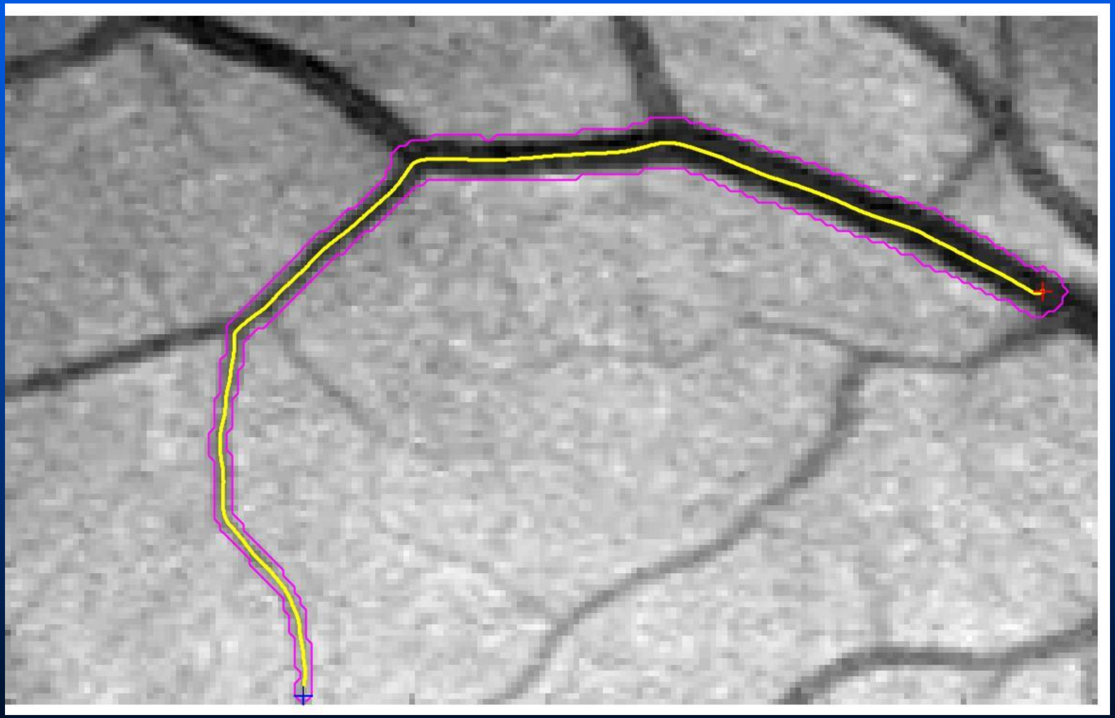
Considers the local orientations of the structures



01/02/2017 19:16 Laurent D. COHEN, Huawei 2017 135

Examples of 3D Minimal Paths for tubular shapes in 2D

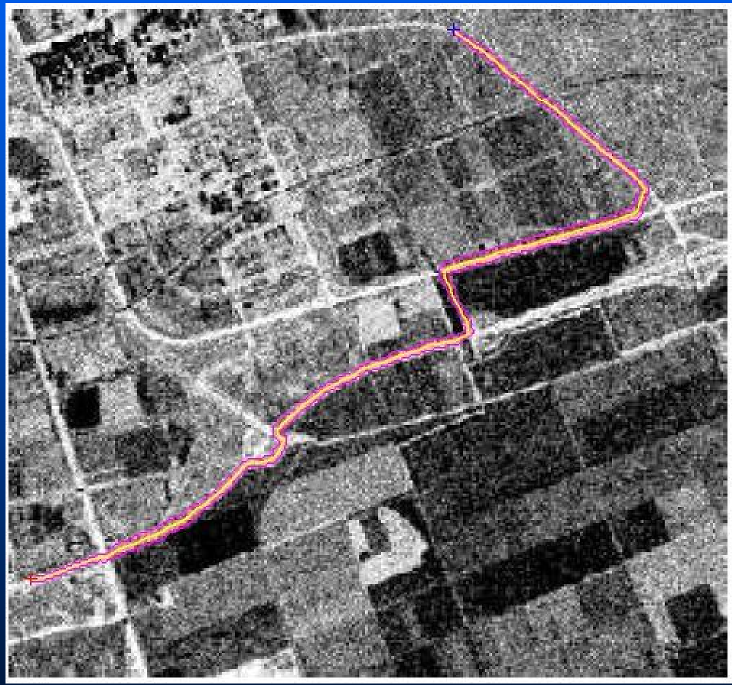
2D in space , 1D for radius of vessel



01/02/2017 19:16 Laurent D. COHEN, Huawei 2017 136

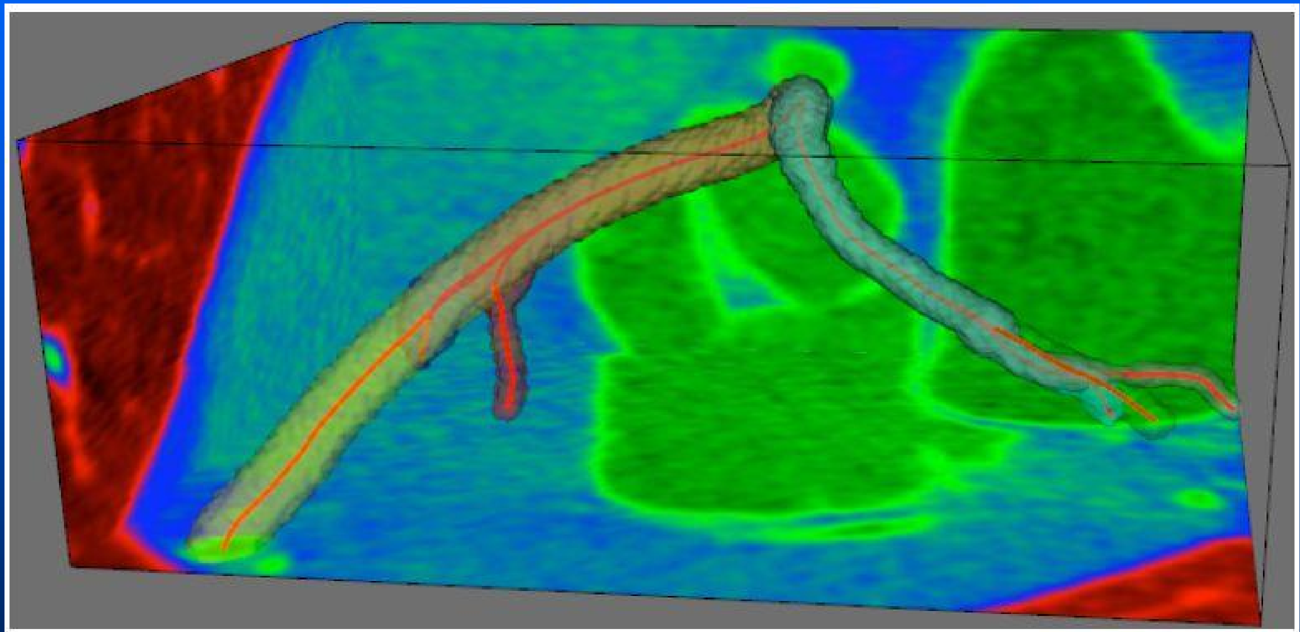
Examples of 3D Minimal Paths for tubular shapes in 2D

2D in space , 1D for radius of vessel



01/02/2017 19:16 Laurent D. COHEN, Huawei 2017 139

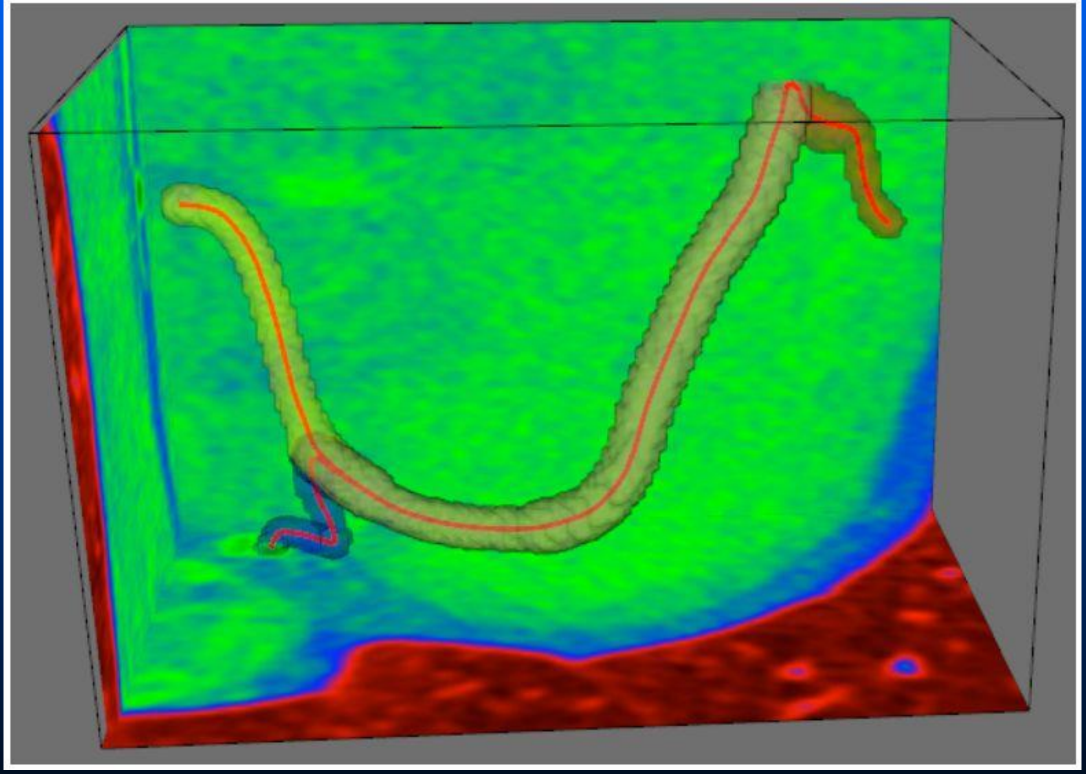
Examples of 4D Minimal Paths for tubular shapes in 3D



01/02/2017 19:16 Laurent D. COHEN, Huawei 2017 141

Examples of 4D Minimal Paths for tubular shapes in 3D

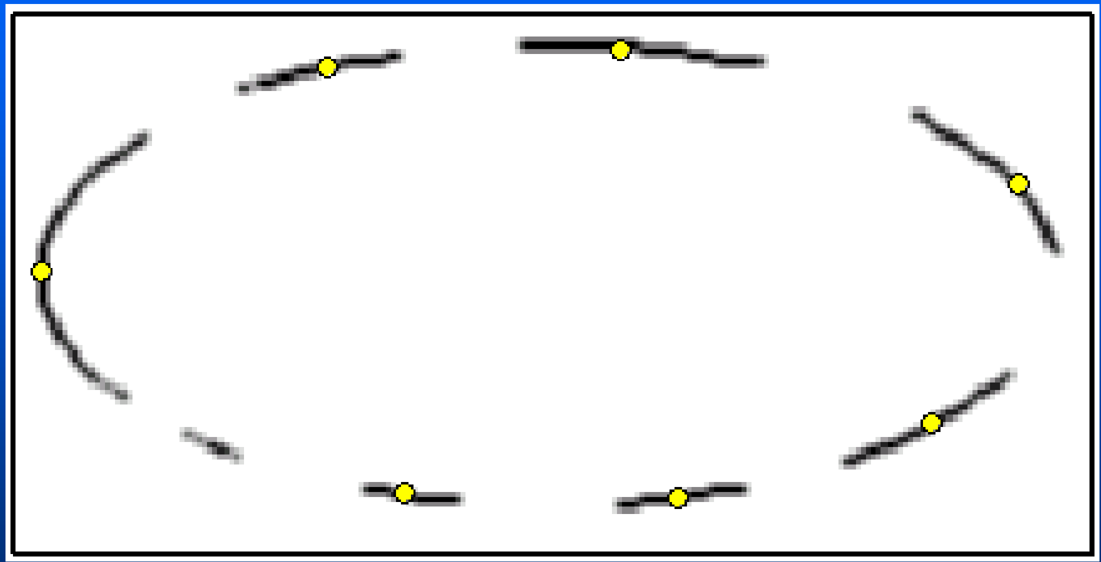
3D in space , 1D for radius of vessel



01/02/2017 19:16 Laurent D. COHEN, Huawei 2017 142

Perceptual Grouping using Minimal Paths

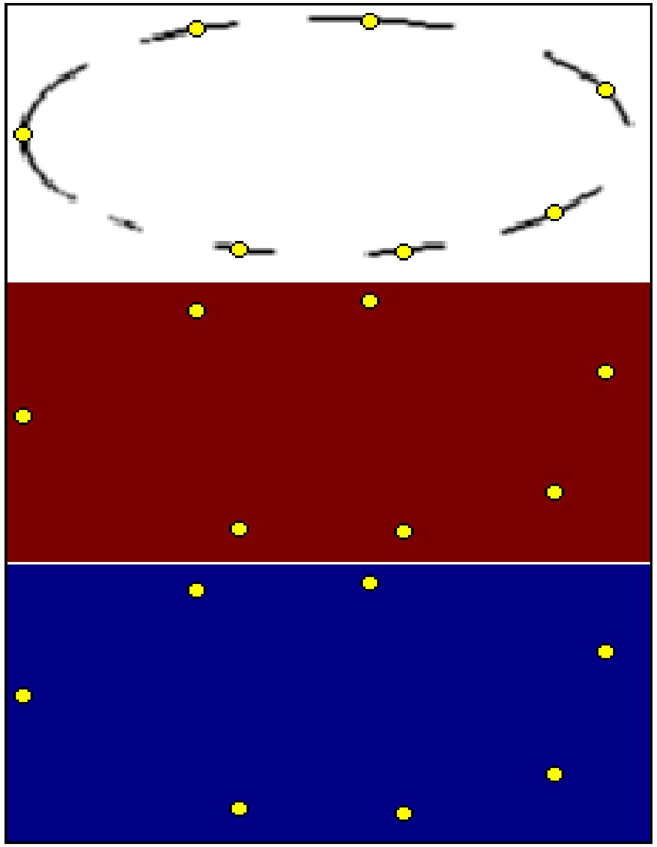
The potential is an incomplete ellipse and 7 points are given (keypoints were found using a Furthest point strategy).

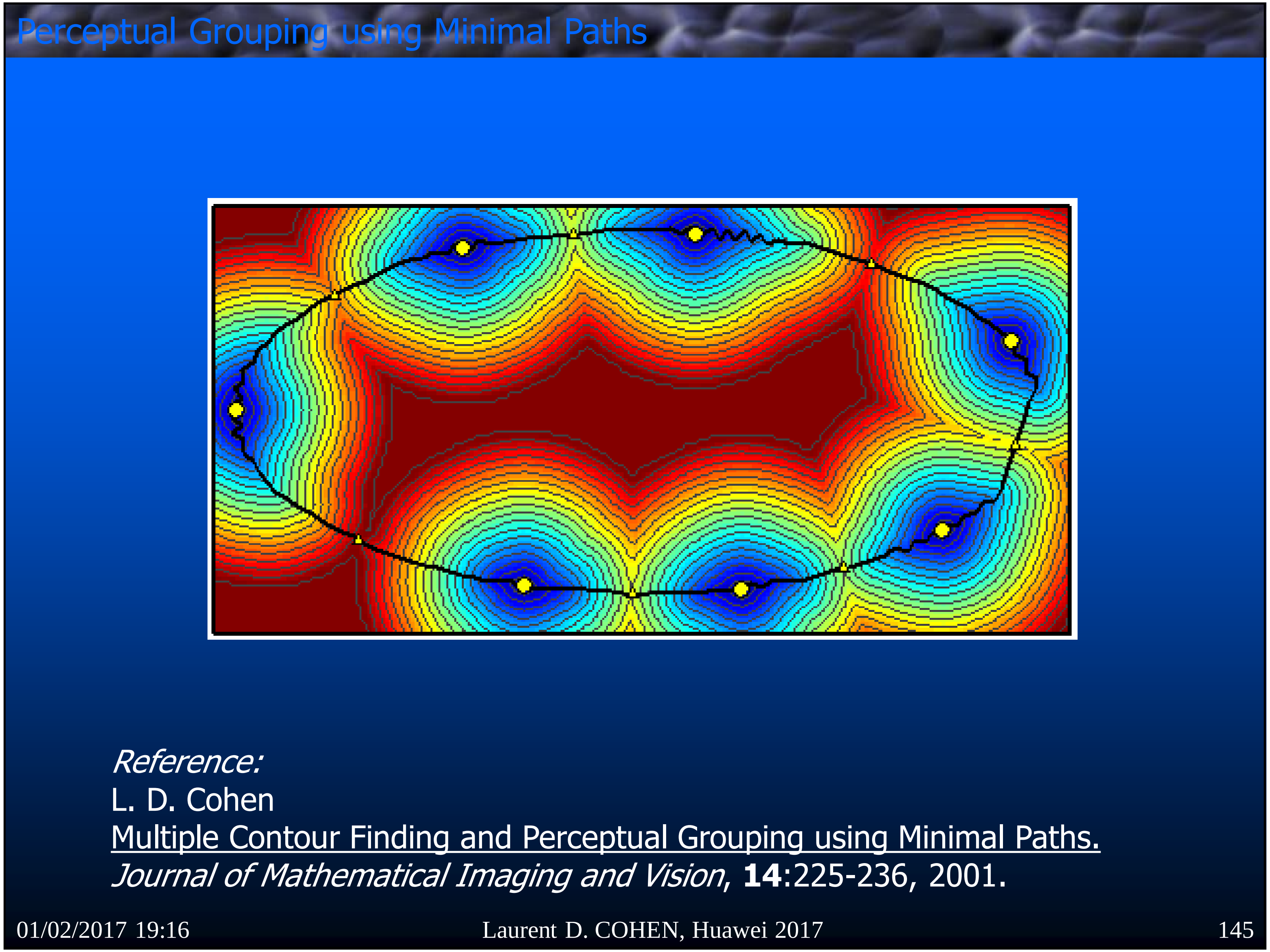


Reference:
L. D. Cohen
Multiple Contour Finding and Perceptual Grouping using Minimal Paths.
Journal of Mathematical Imaging and Vision, **14**:225-236, 2001.

01/02/2017 19:16 Laurent D. COHEN, Huawei 2017 143

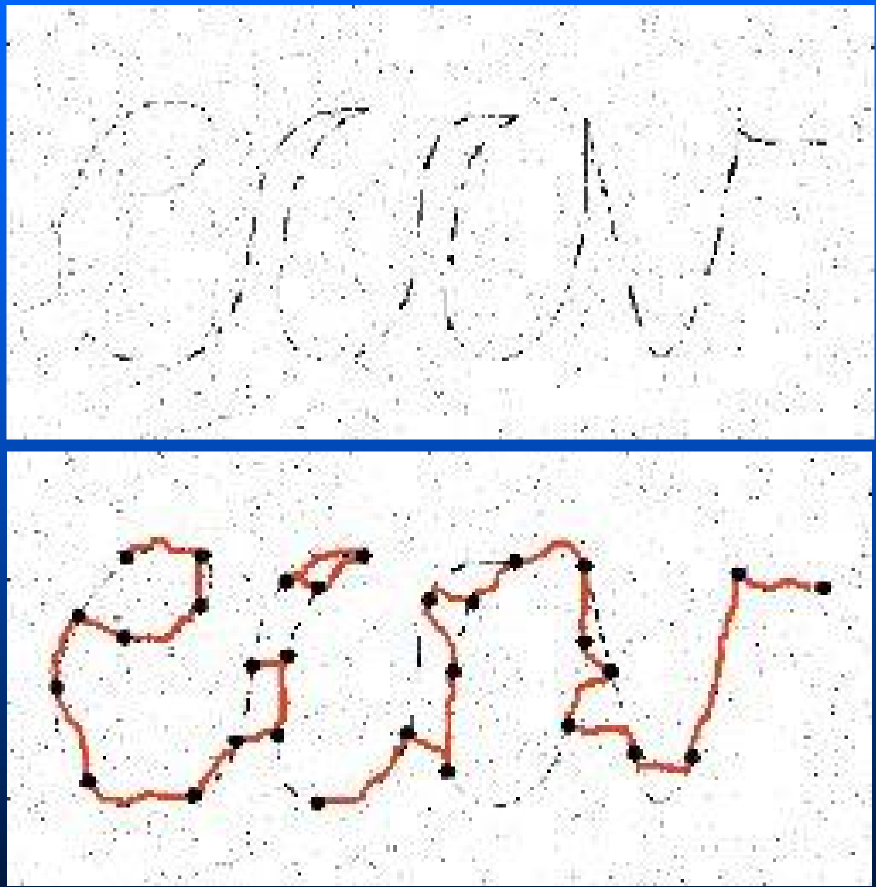
Perceptual Grouping using a set of Minimal Paths




Perceptual Grouping using Minimal Paths

Reference:
L. D. Cohen
Multiple Contour Finding and Perceptual Grouping using Minimal Paths.
Journal of Mathematical Imaging and Vision, **14**:225-236, 2001.
01/02/2017 19:16
Laurent D. COHEN, Huawei 2017
145

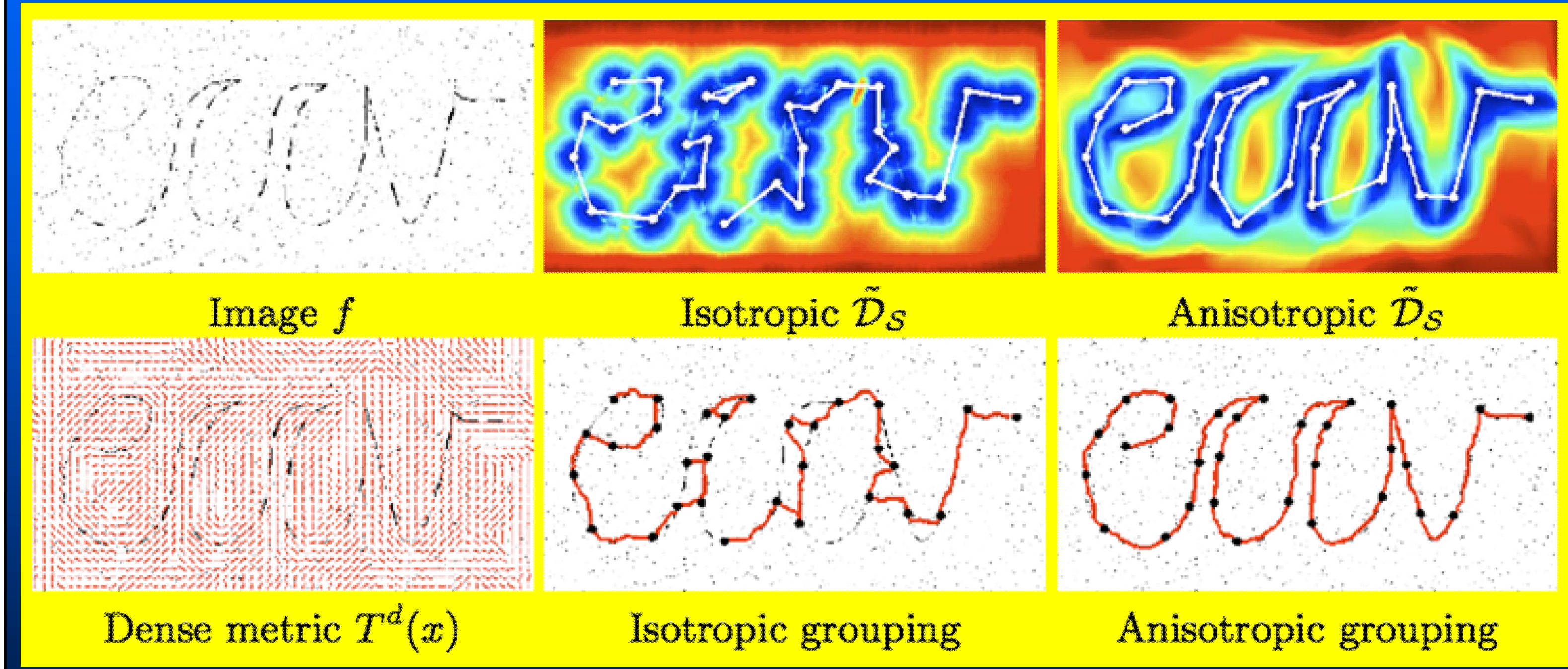
Perceptual Grouping using Minimal Paths



01/02/2017 19:16 Laurent D. COHEN, Huawei 2017 146

Perceptual Grouping using Minimal Paths

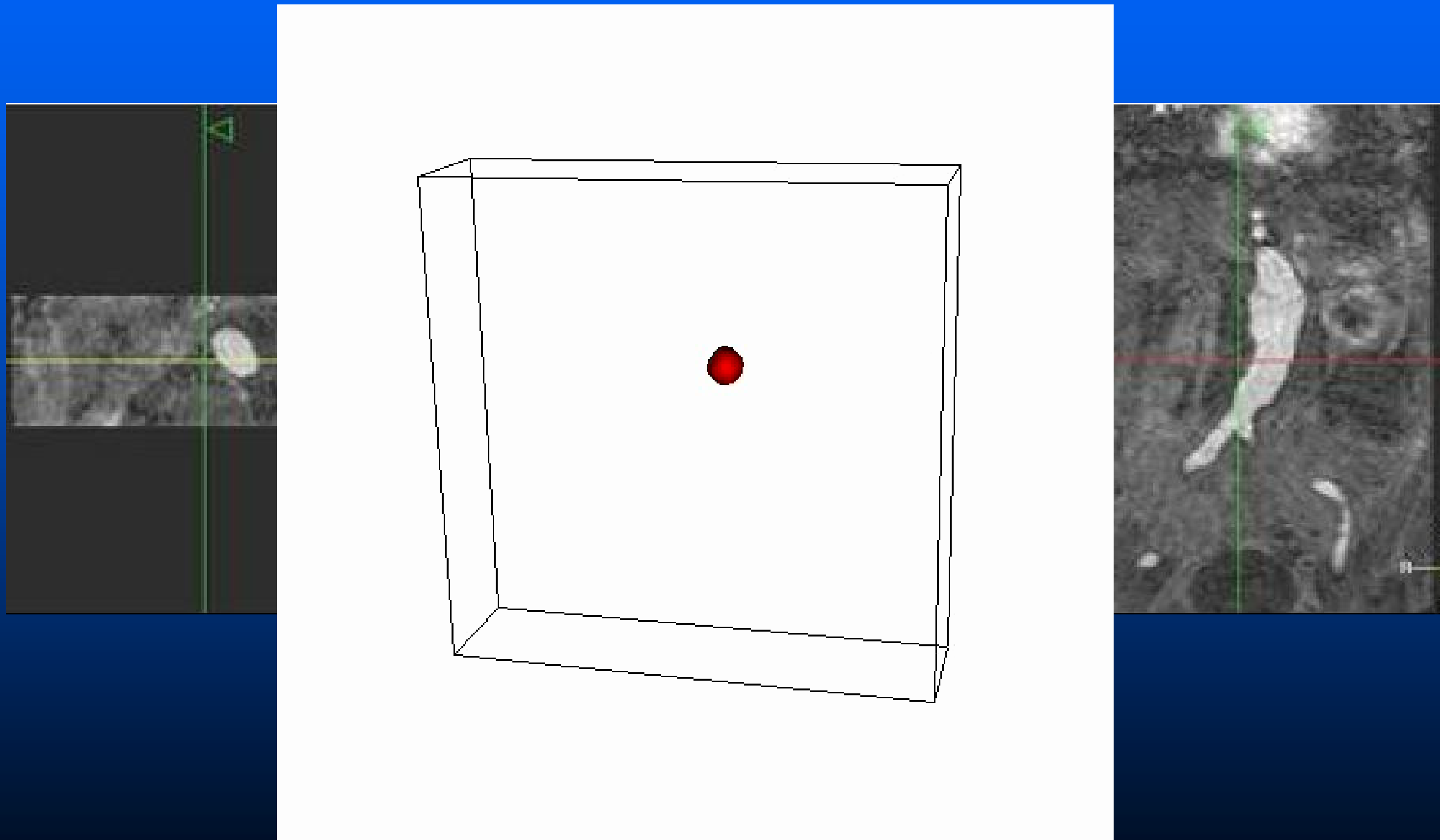
Using the orientation with anisotropic geodesics



Anisotropic Geodesics for Perceptual Grouping and Domain Meshing. Sebastien Bougleux and Gabriel Peyr\'e and Laurent D. Cohen. Proc. tenth European Conference on Computer Vision (ECCV'08)}, Marseille, France, October 12-18, 2008.

01/02/2017 19:16 Laurent D. COHEN, Huawei 2017 147

Application Endoscopie Virtuelle (collaboration Philips Recherche)



01/02/2017 19:16 Laurent D. COHEN, Huawei 2017 148

Vue Endoscopique dâun arbre vasculaire

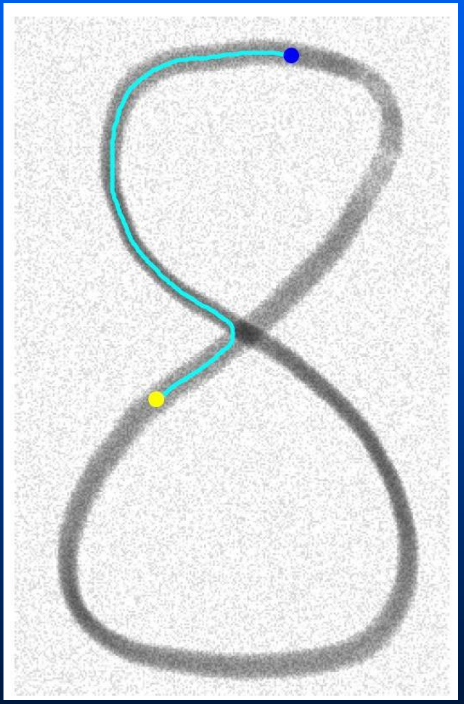
01/02/2017 19:16 Laurent D. COHEN, Huawei 2017 149

Curvature Penalized Minimal Path Method with A Finsler Metric

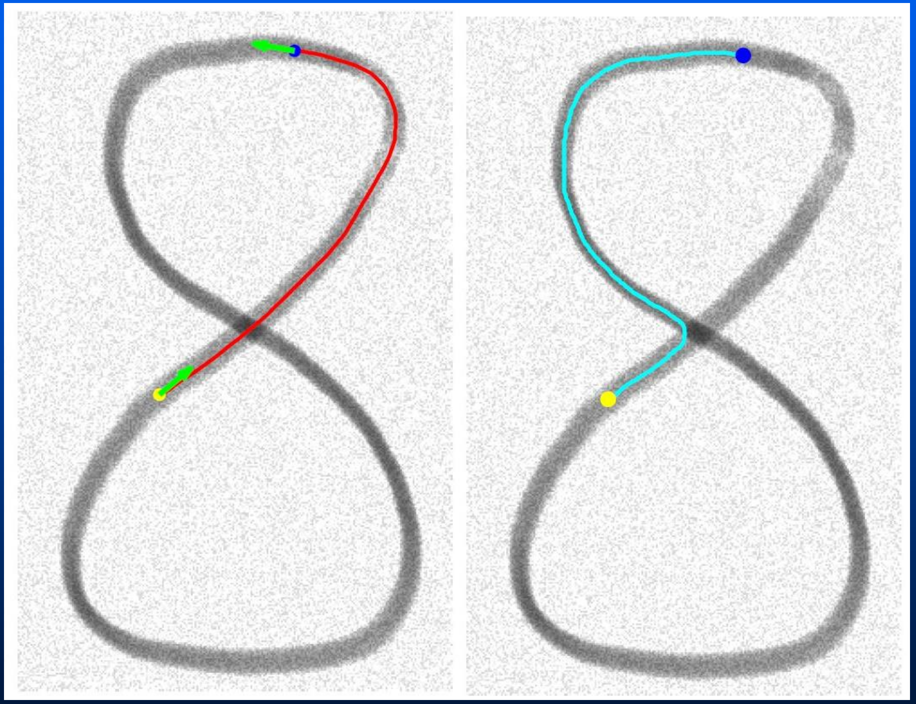
with Da Chen and JM Mirebeau, 2015-2016

- The metric may depend on the orientation
- Orientation-lifted metric: the curve length of Euler elastica can be exactly computed by this metric

Curvature Penalized Minimal Path Method with A Finsler Metric

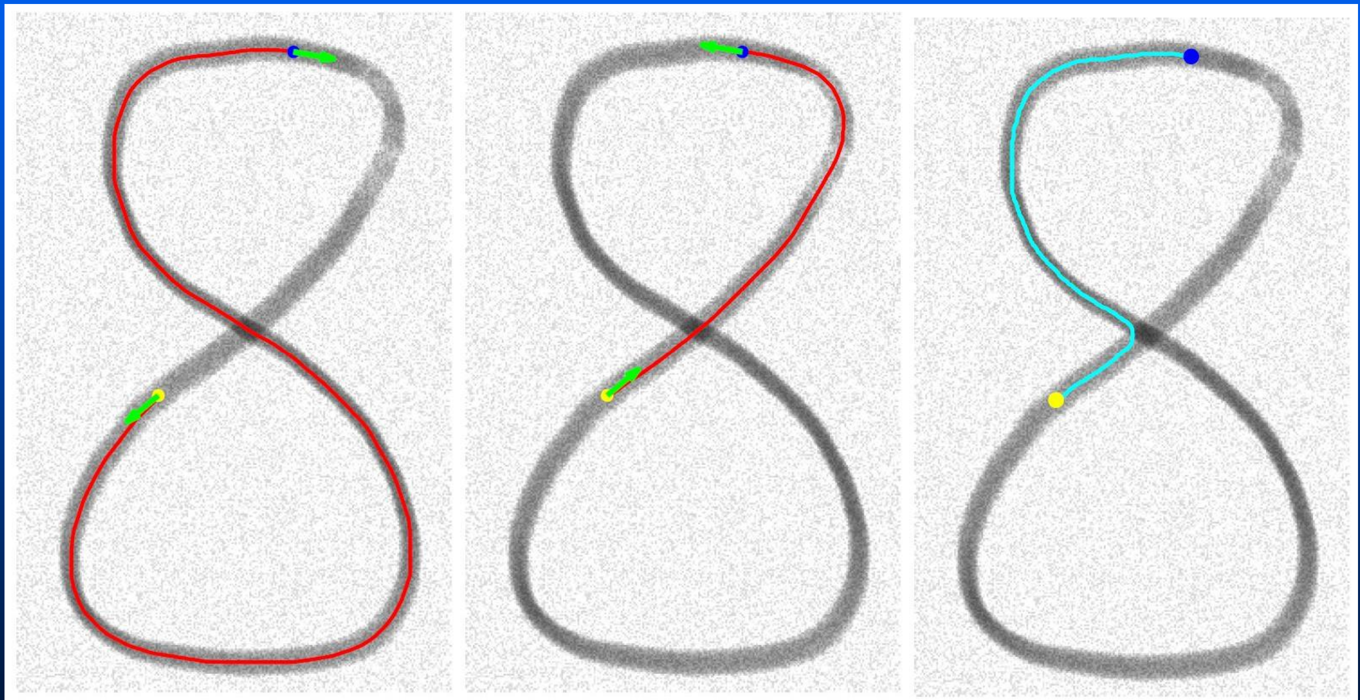


Curvature Penalized Minimal Path Method with A Finsler Metric



01/02/2017 19:16 Laurent D. COHEN, Huawei 2017 157

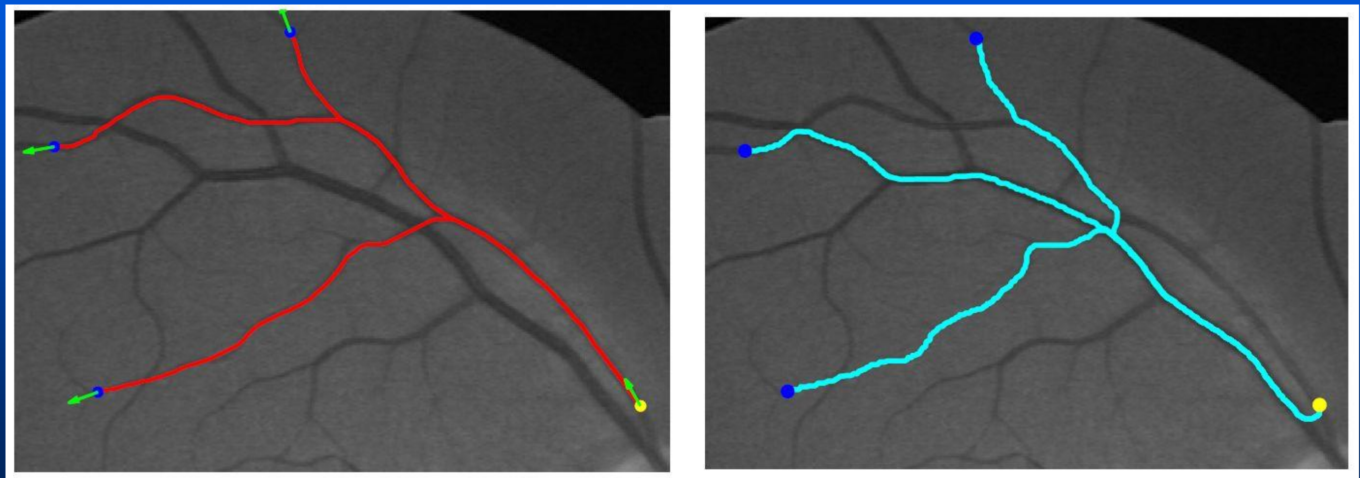
Curvature Penalized Minimal Path Method with A Finsler Metric



01/02/2017 19:16 Laurent D. COHEN, Huawei 2017 158

Curvature Penalized Minimal Path Method with A Finsler Metric

Fig. 8 Geodesics extraction results using the proposed Finsler metric. Red and green dots are the initial and end positions respectively. Arrows indicate the corresponding tangents.



Curvature Penalized Minimal Path Method with A Finsler Metric

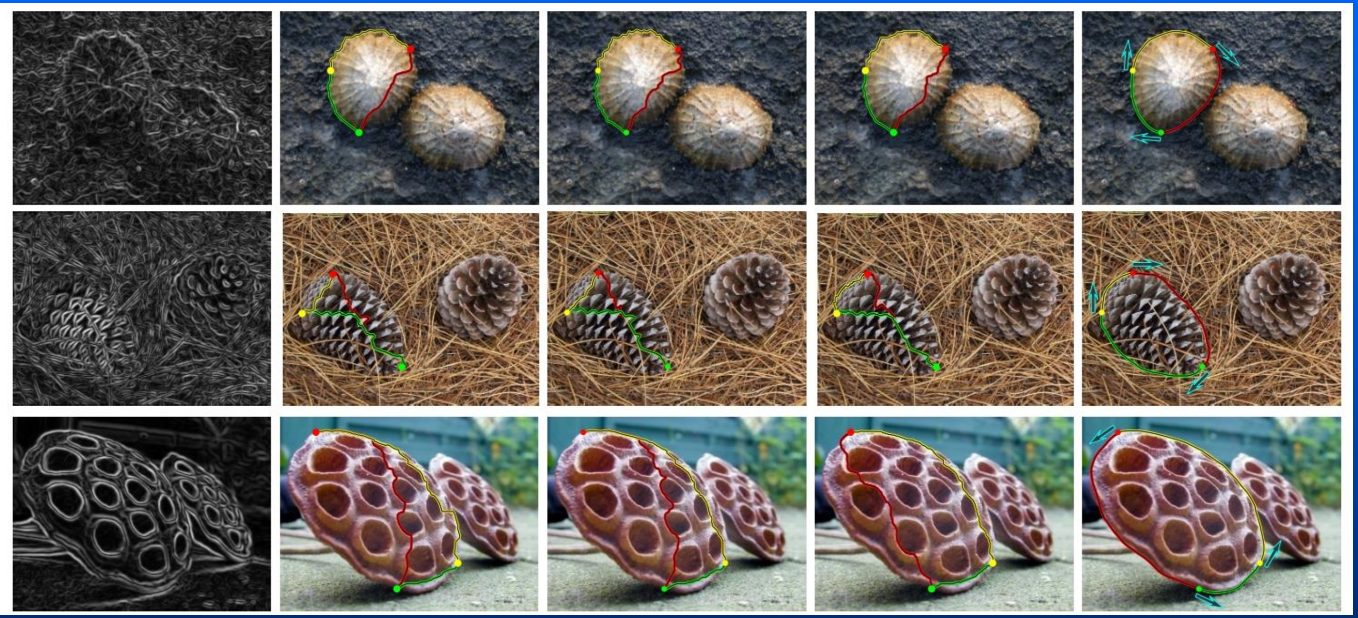


Fig. 9 Comparative closed contour detection results. **Column 1**: edge saliency map. **Columns 2-5**: results from the IR metric, the AR metric, the IOLR metric and the proposed Finsler metric. In Column 5, arrows indicate the tangents for the corresponding physical positions denoted by dots.

Curvature Penalized Minimal Path Method with A Finsler Metric

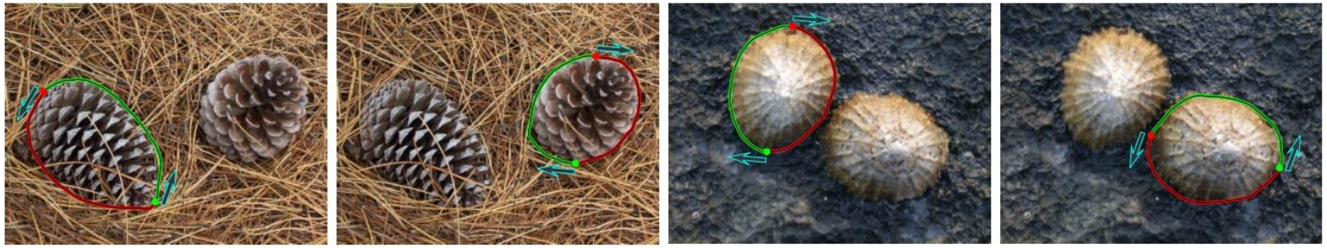
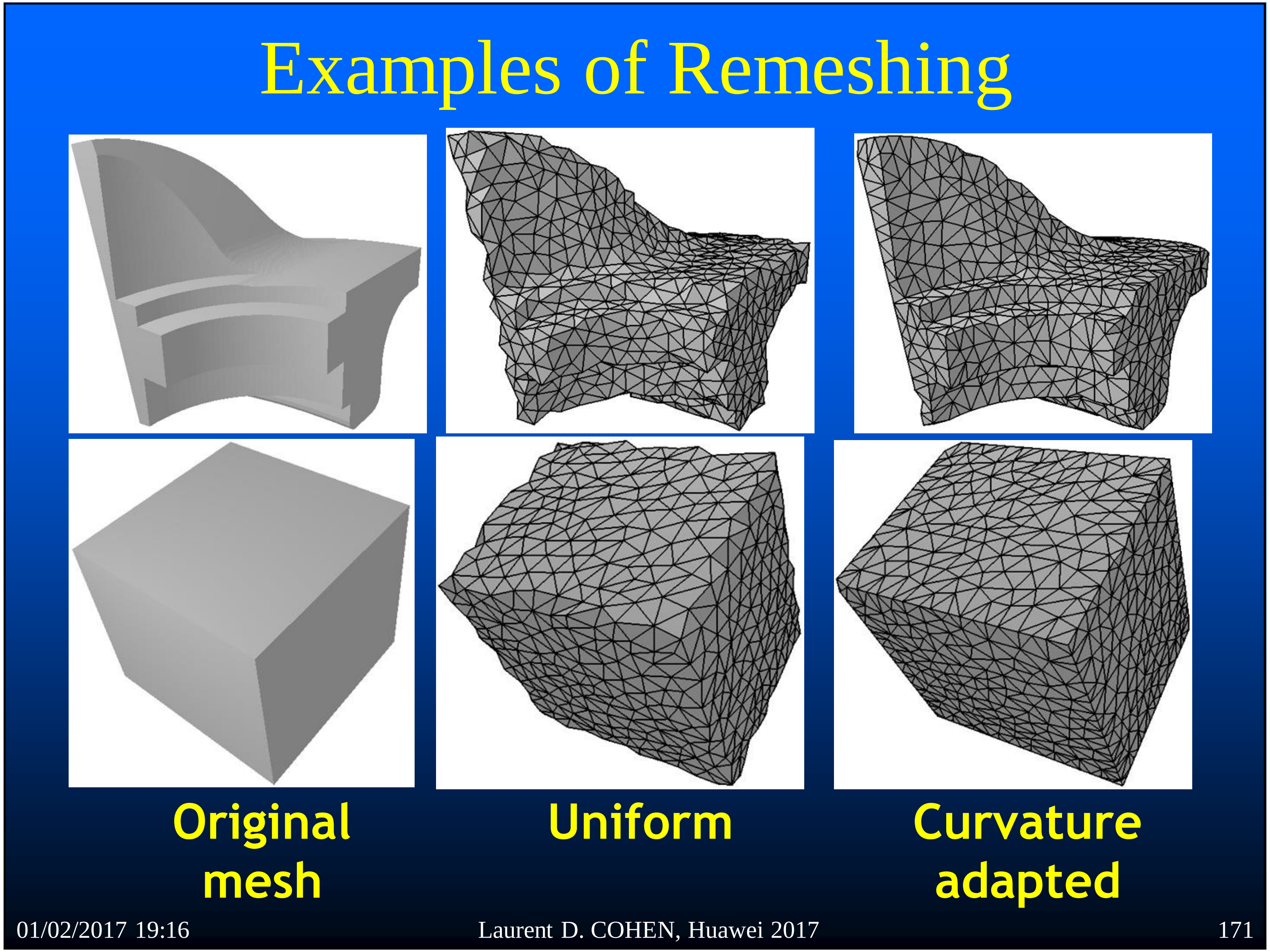
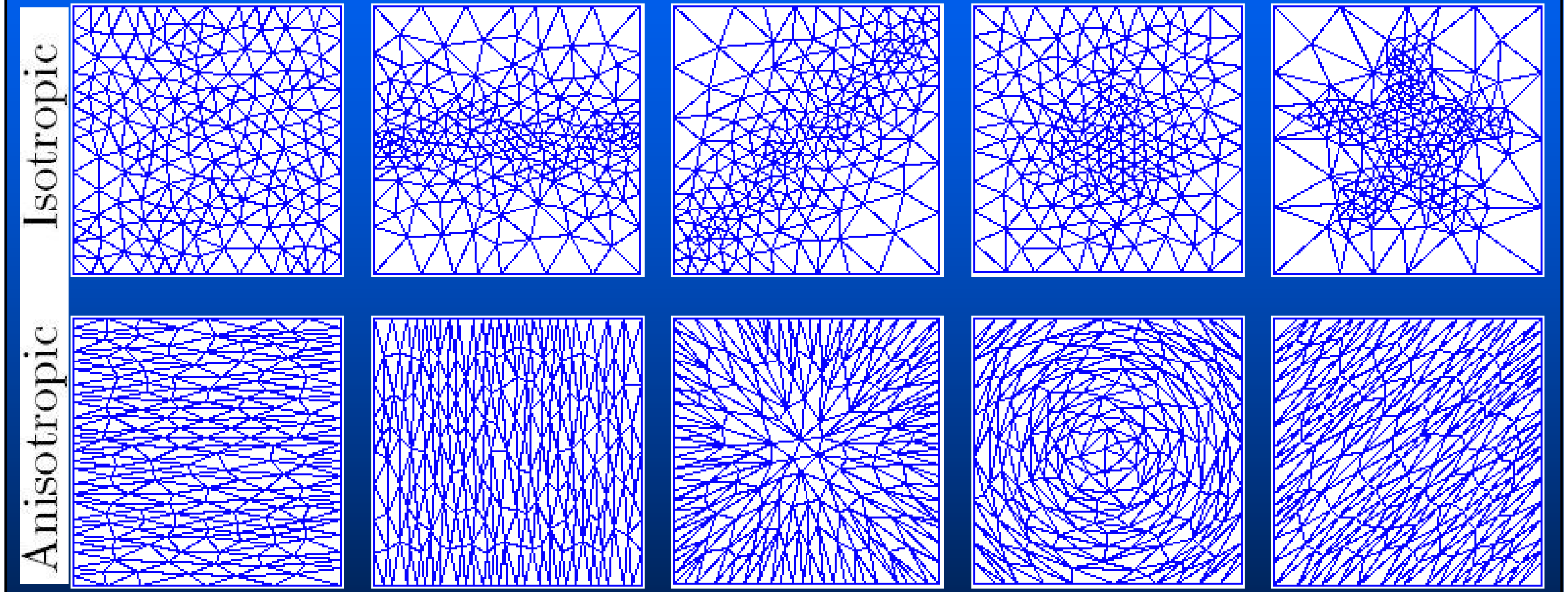


Fig. 10 Closed contour detection results using only two given physical positions.

01/02/2017 19:16 Laurent D. COHEN, Huawei 2017 167

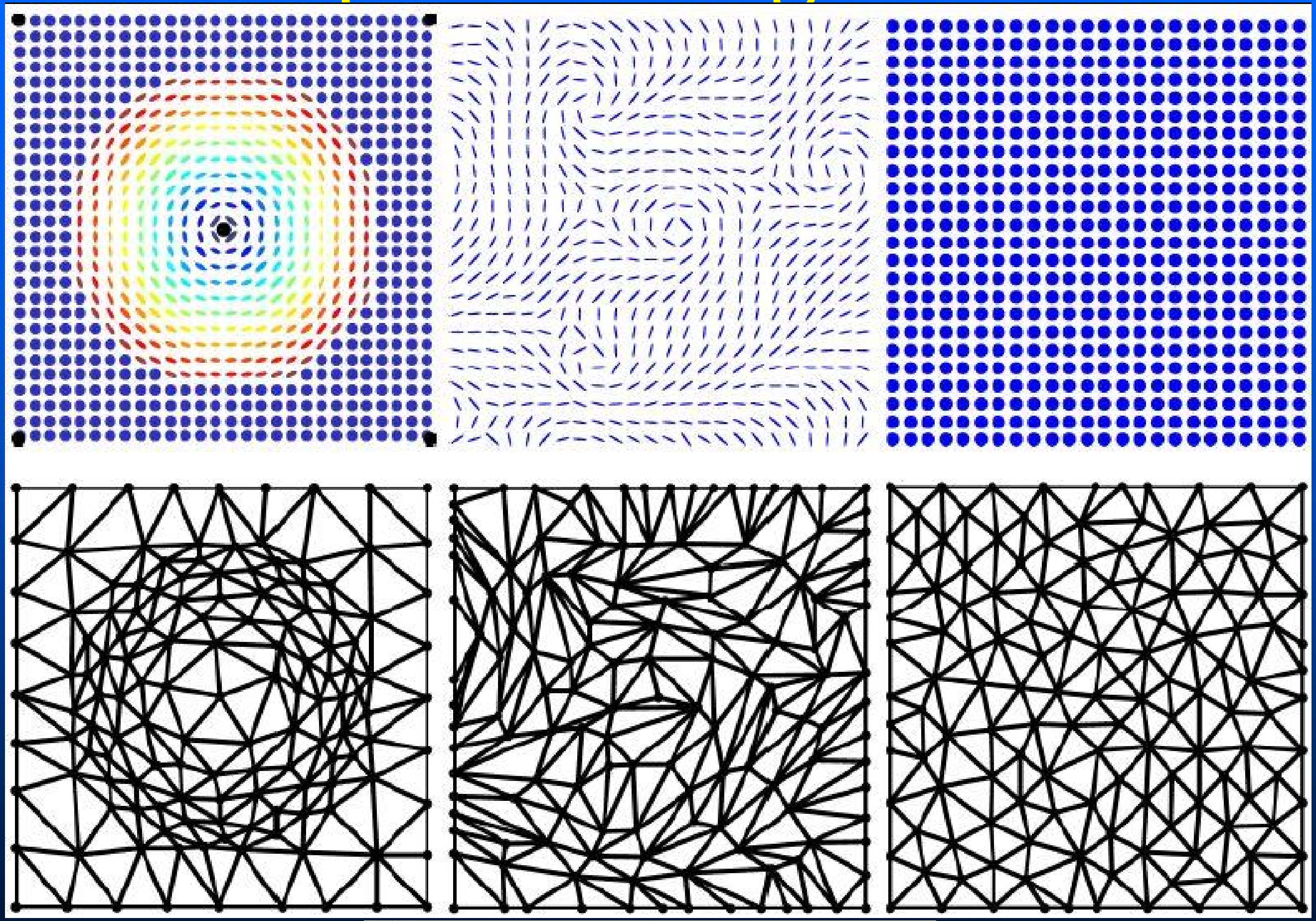

Examples of Remeshing
Original mesh
Uniform
Curvature adapted
01/02/2017 19:16
Laurent D. COHEN, Huawei 2017
171

Isotropic vs. Anisotropic Meshing



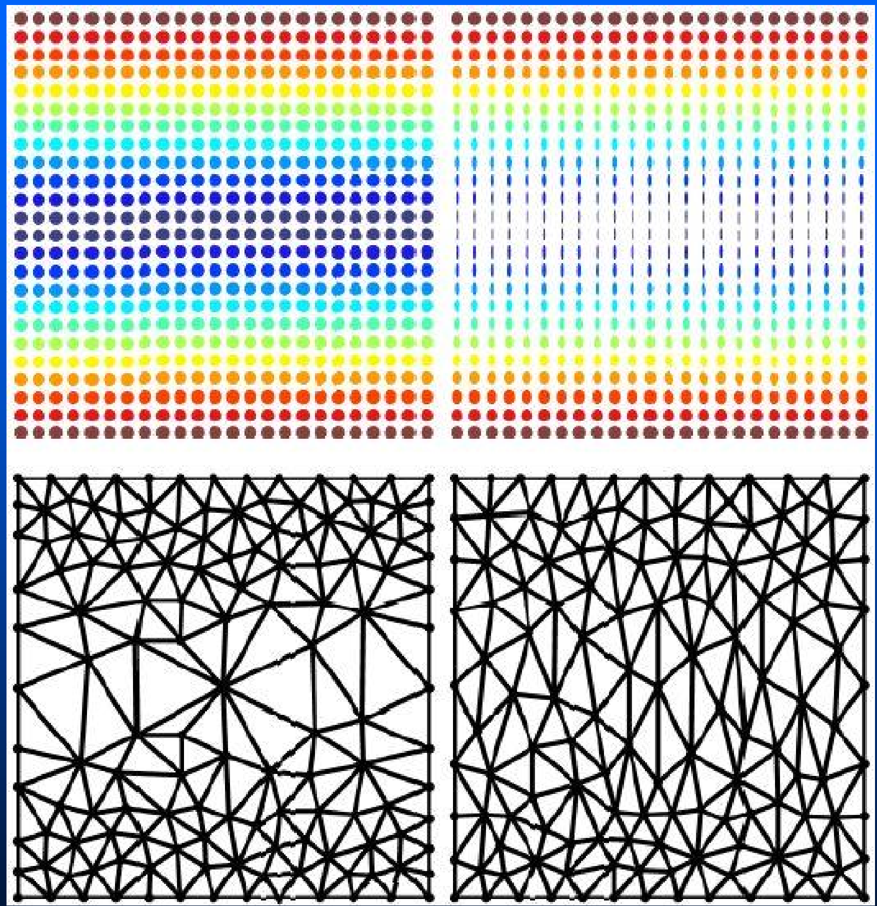
01/02/2017 19:16 Laurent D. COHEN, Huawei 2017 172

Anisotropic Meshing



farthest point strategy

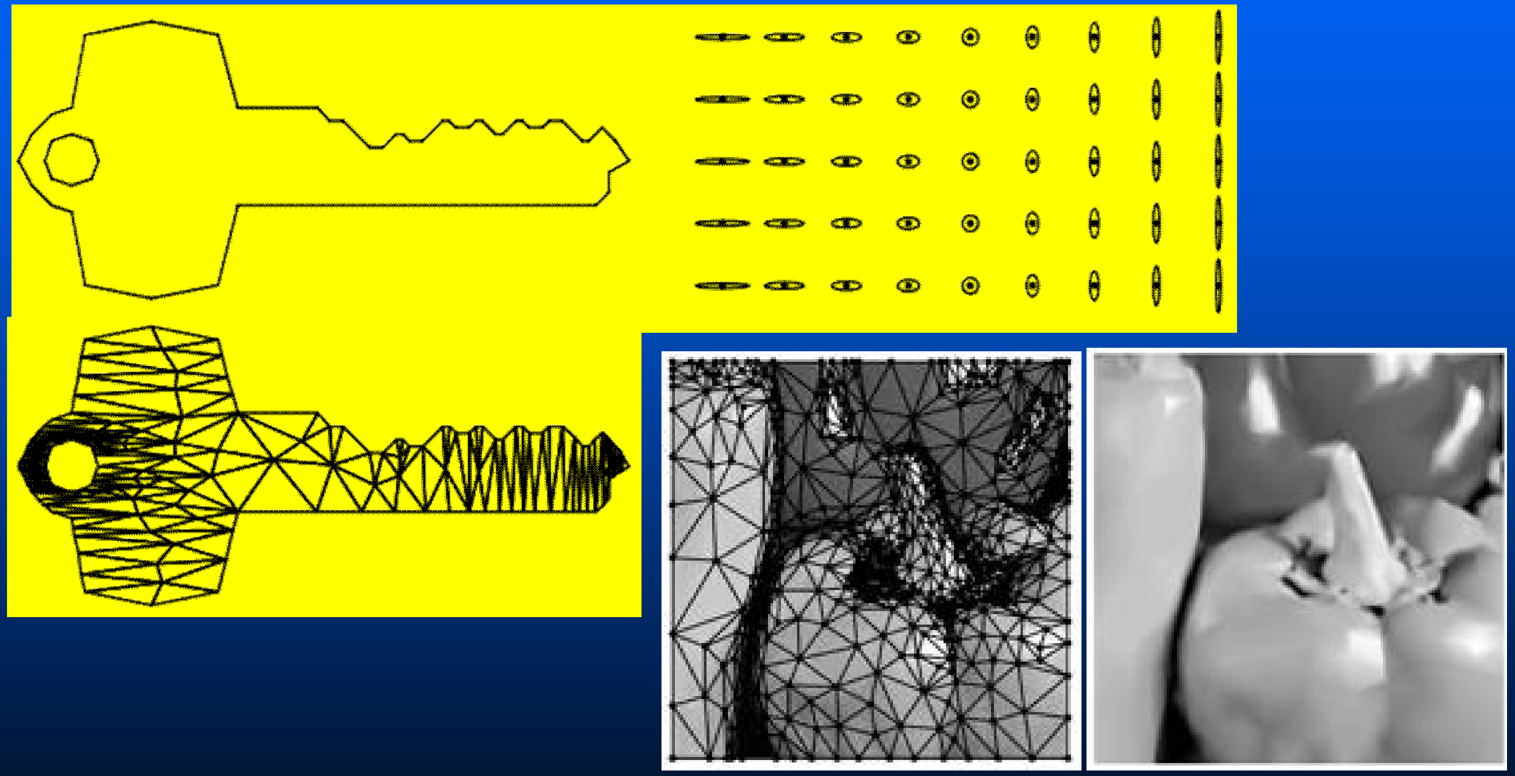
Anisotropic Meshing



farthest point strategy

Examples of Anisotropic Meshing

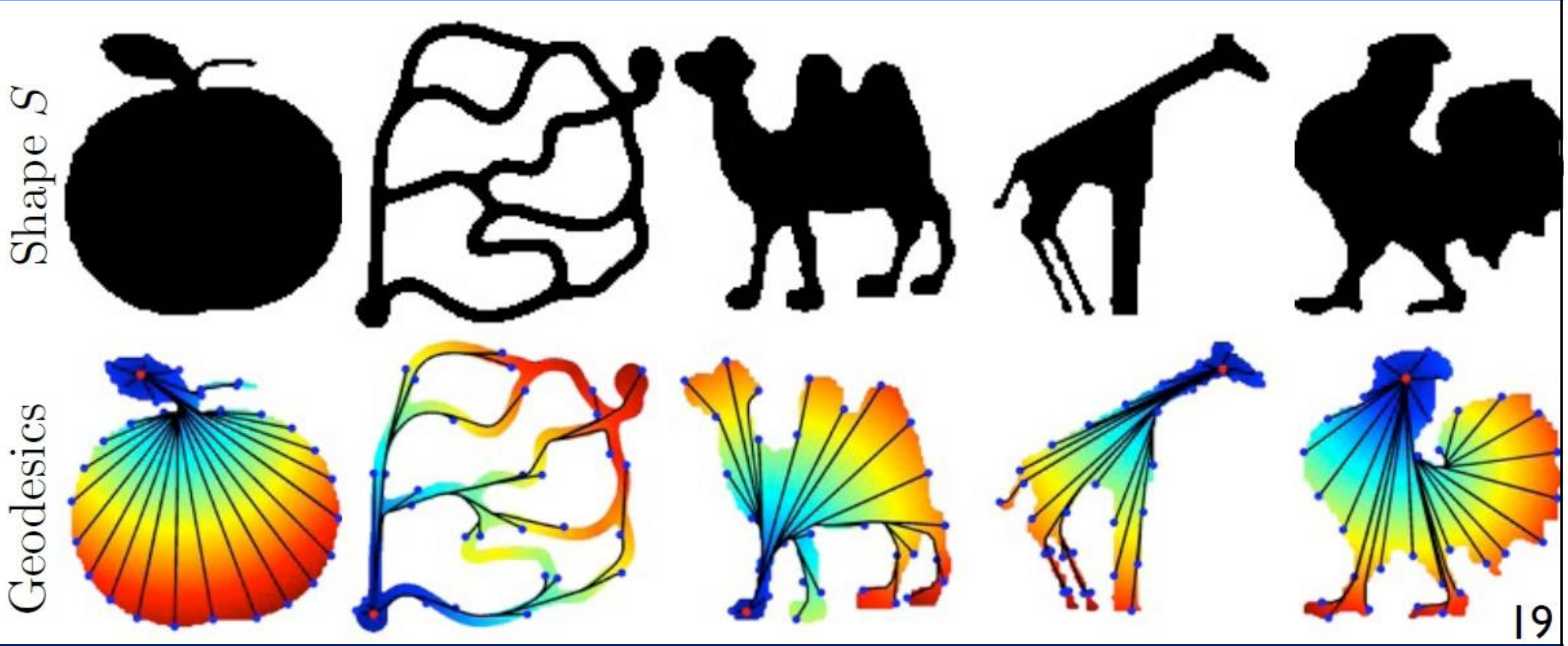
controls density and orientation of triangles



01/02/2017 19:16 Laurent D. COHEN, Huawei 2017 175

Geodesic methods for shape recognition

Based on distribution (histogram) of geodesic distances



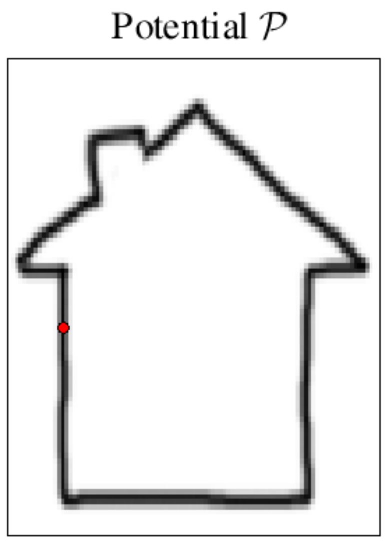
01/02/2017 19:16 Laurent D. COHEN, Huawei 2017 176

Overview

- Minimal Paths, Fast Marching and Front Propagation
- Anisotropic Minimal Paths and Tubular model
- Finding contours as a set of minimal paths
- Application to 2D and 3D tree structures
- Geodesic Density for tree structures

01/02/2017 19:16 Laurent D. COHEN, Huawei 2017 179

Finding a closed contour by growing minimal paths and adding keypoints



[Finding a Closed Boundary by Growing Minimal Paths from a Single Point on 2D or 3D Images]. Fethallah Benmansour and Laurent D. Cohen. Journal of Mathematical Imaging and Vision. 2009.

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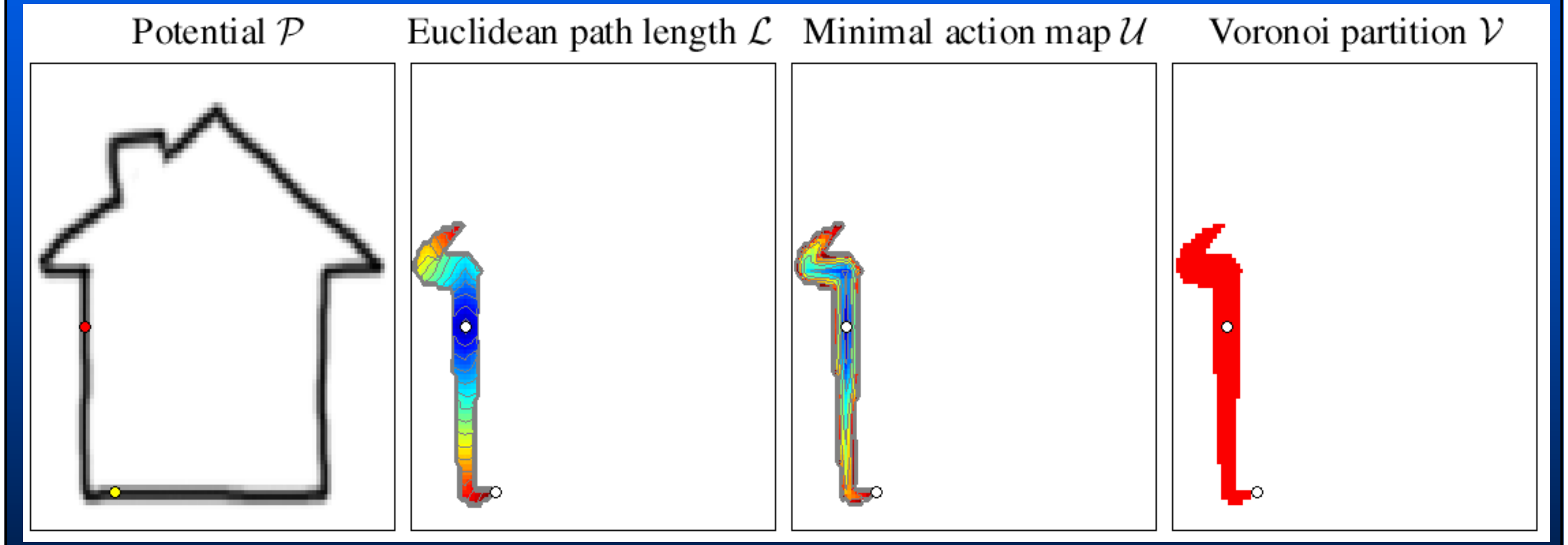
Finding a closed contour by growing minimal paths and adding keypoints



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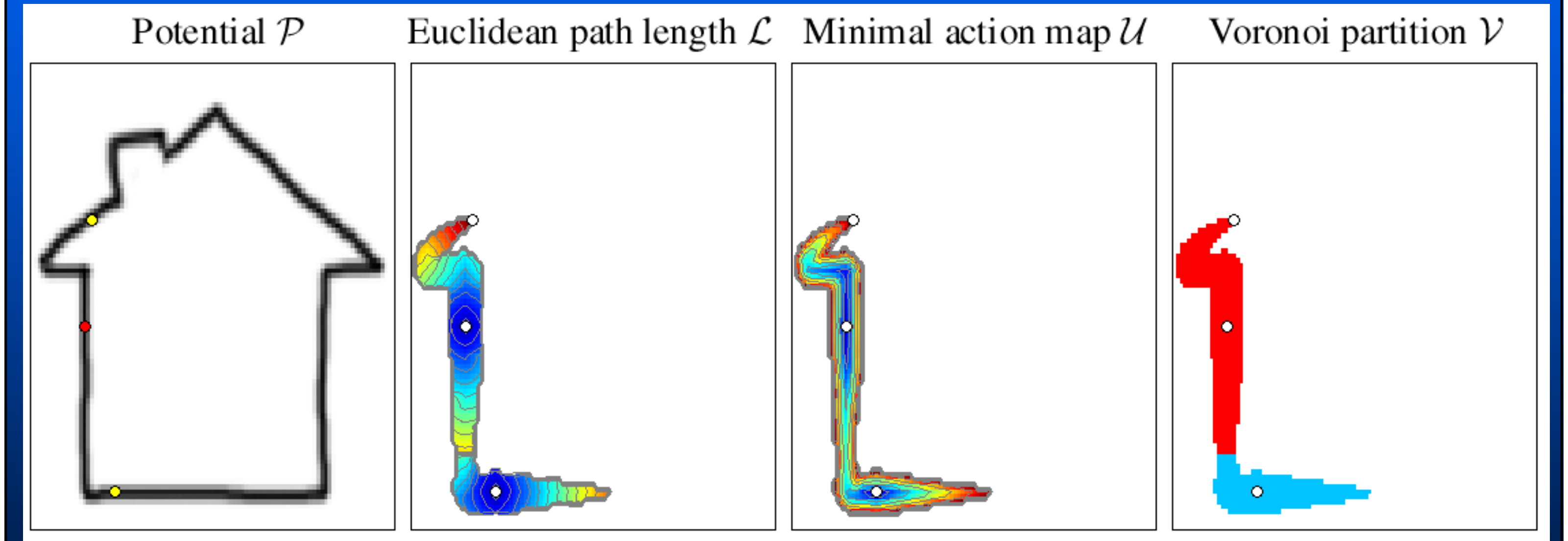
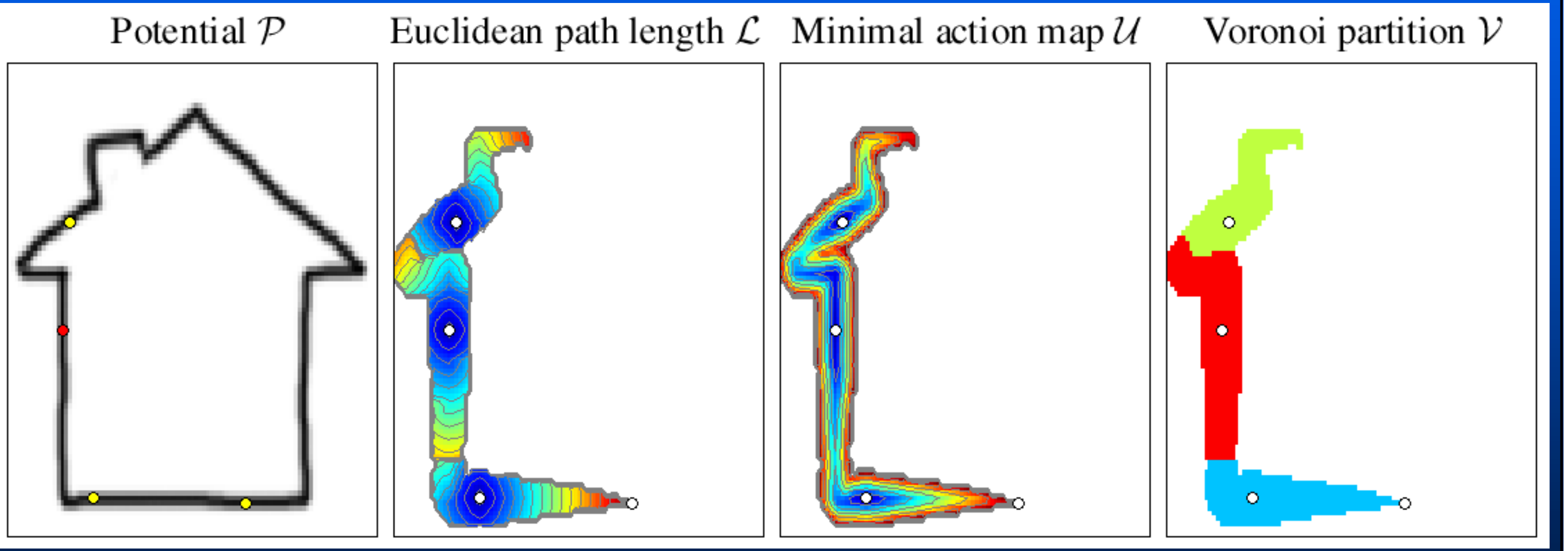
Finding a closed contour by growing minimal paths and adding keypoints

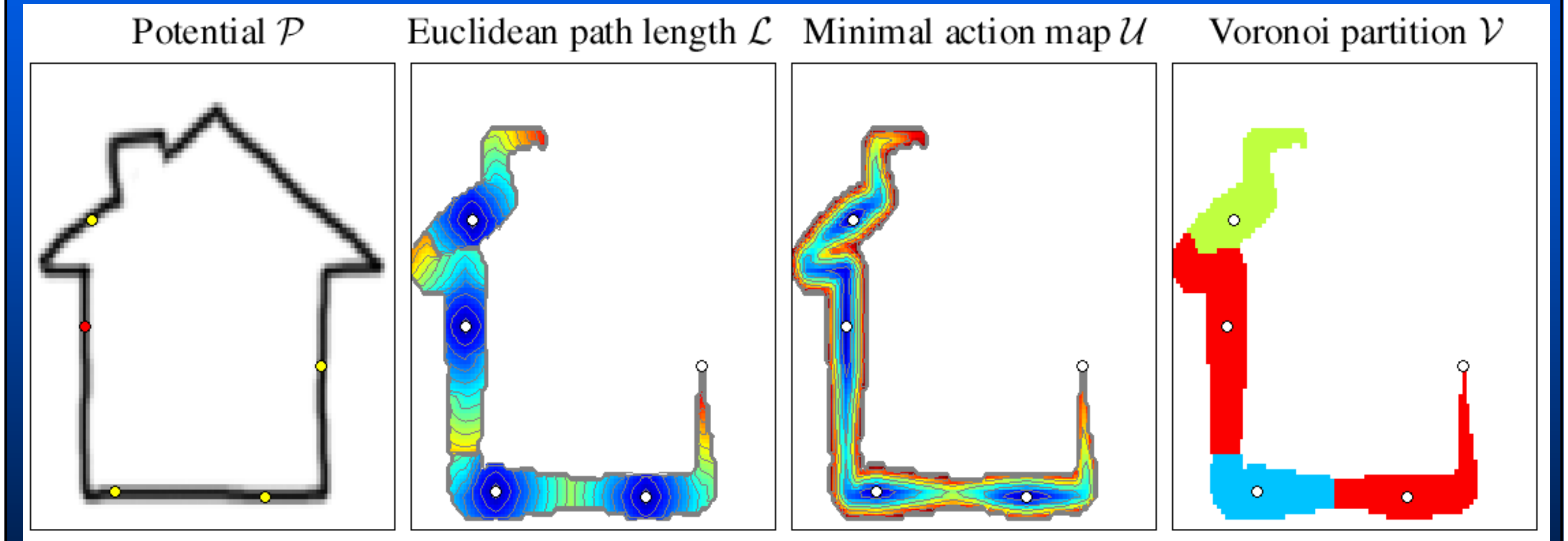


01/02/2017 19:16 Laurent D. COHEN, Huawei 2017 187

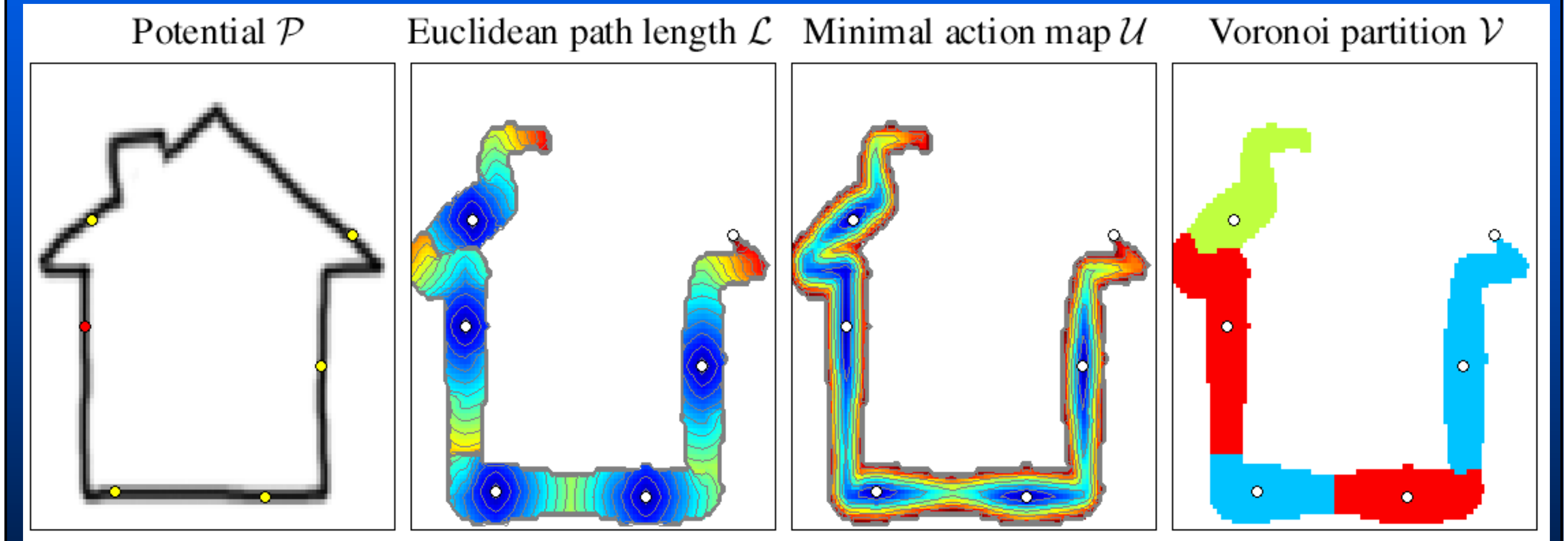
Finding a closed contour by growing minimal paths and adding keypoints



Finding a closed contour by growing minimal paths and adding keypoints



Finding a closed contour by growing minimal paths and adding keypoints



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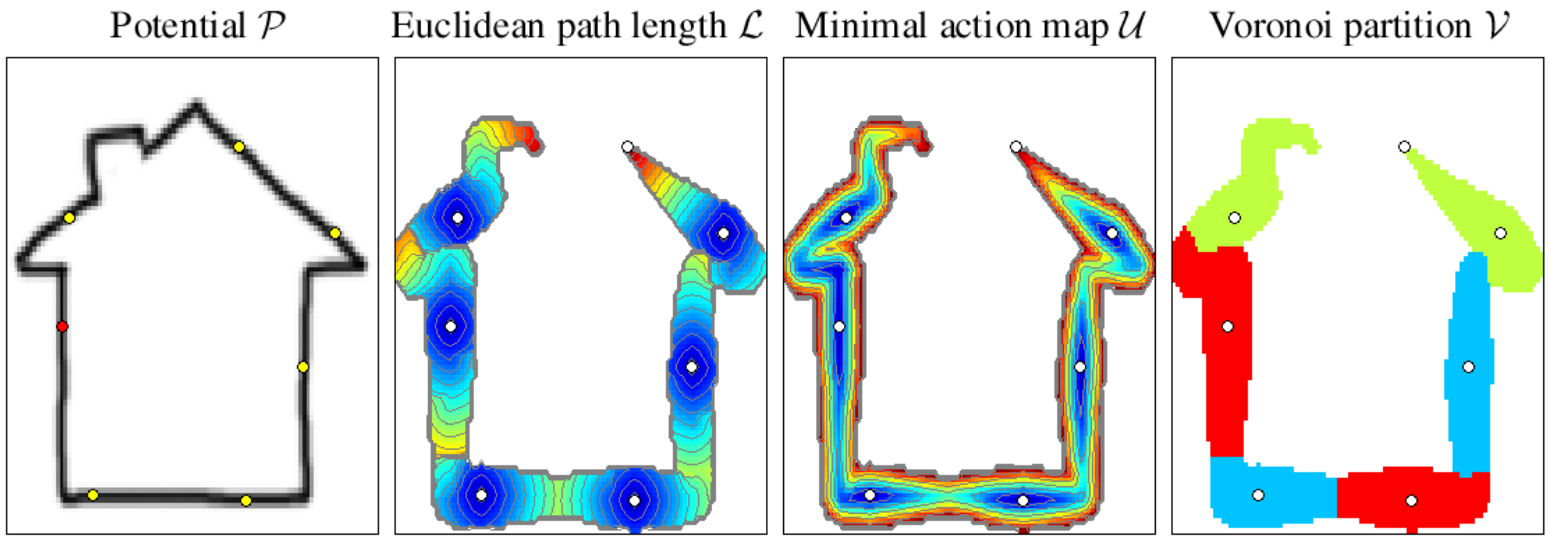
Finding a closed contour by growing minimal paths and adding keypoints



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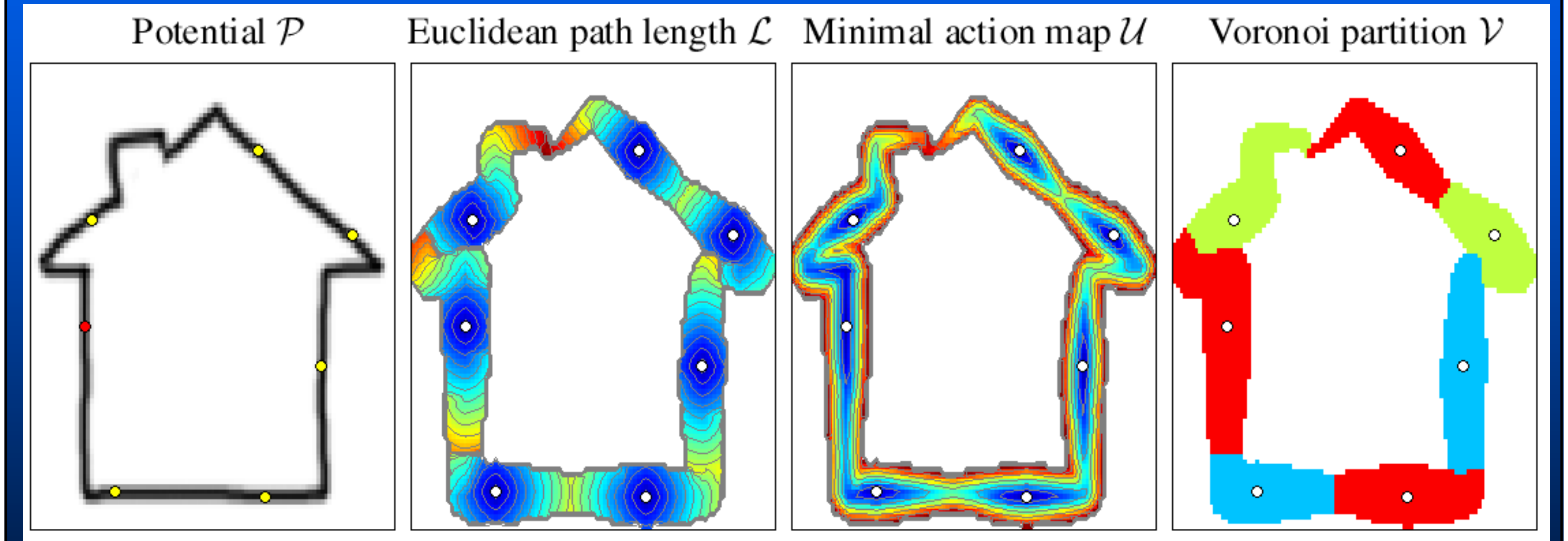
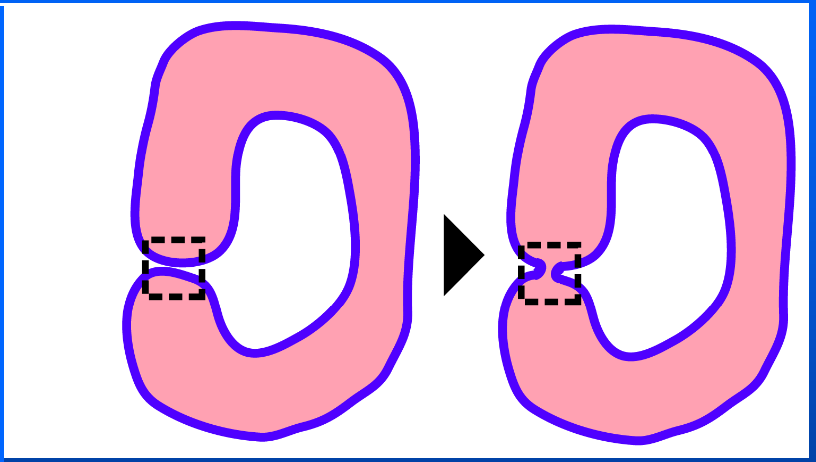
Finding a closed contour by growing minimal paths and adding keypoints



01/02/2017 19:16 Laurent D. COHEN, Huawei 2017 192

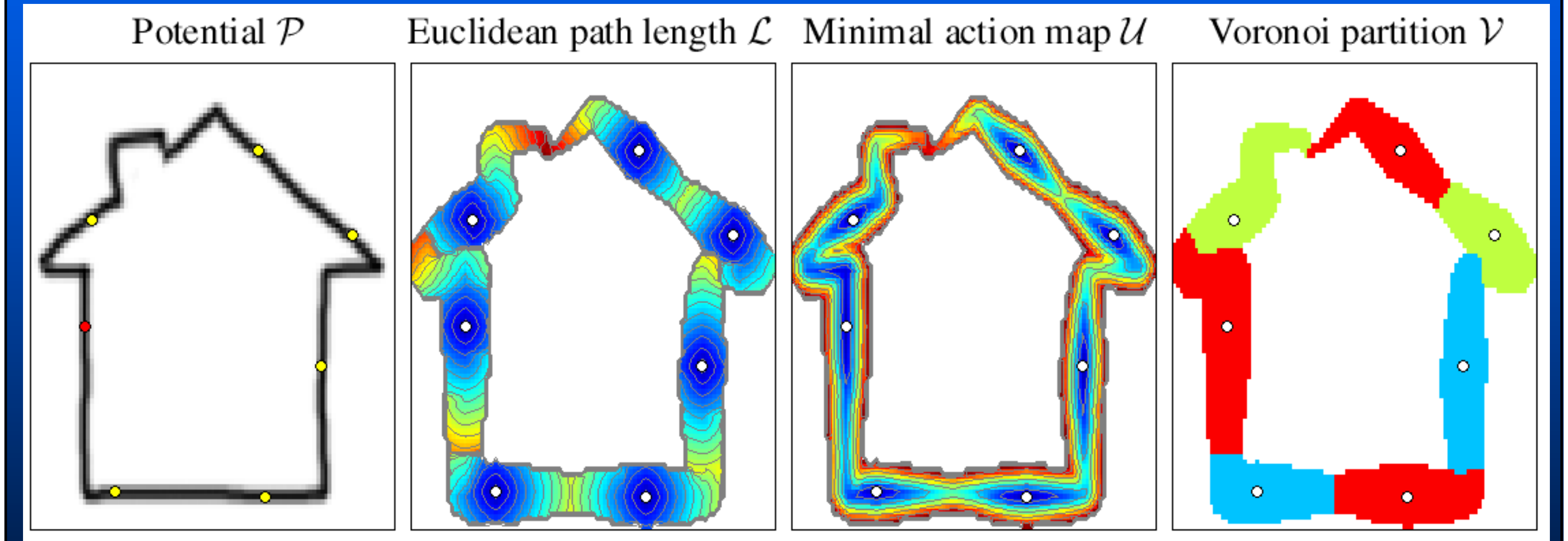
Adding keypoints: Stopping criterion

The propagation must be stopped as soon as the domain visited by the fronts has the same topology as a ring.

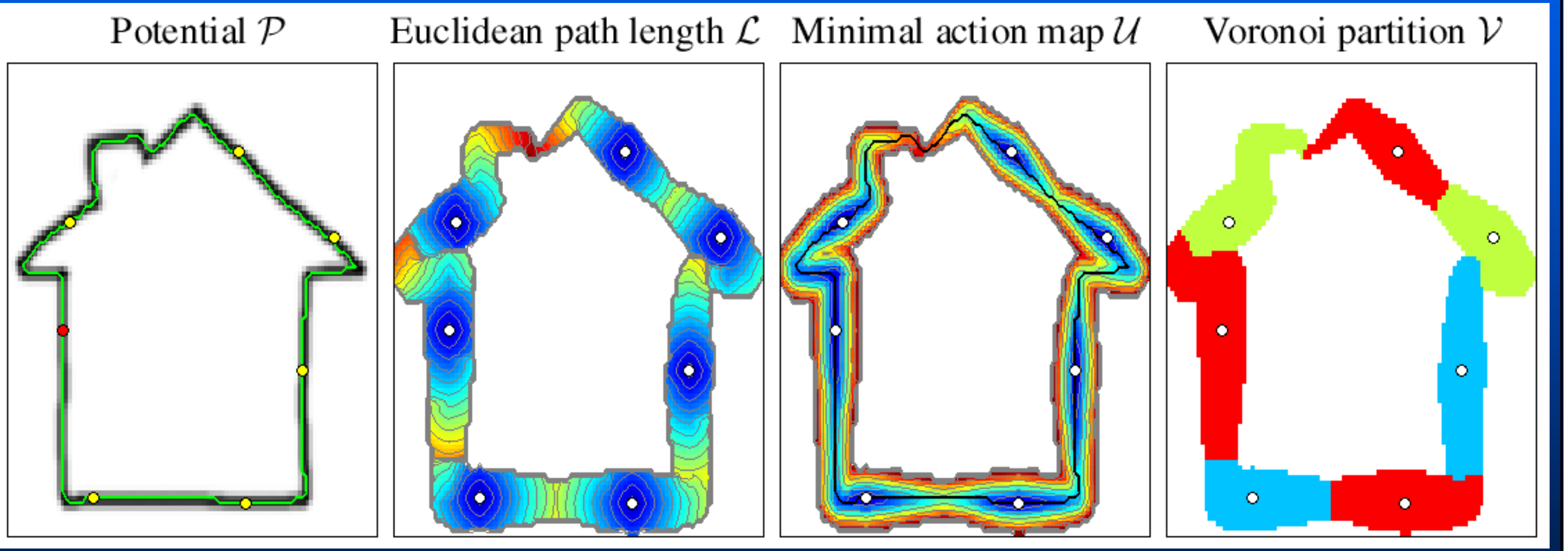


01/02/2017 19:16 Laurent D. COHEN, Huawei 2017 193

Finding a closed contour by growing minimal paths and adding keypoints

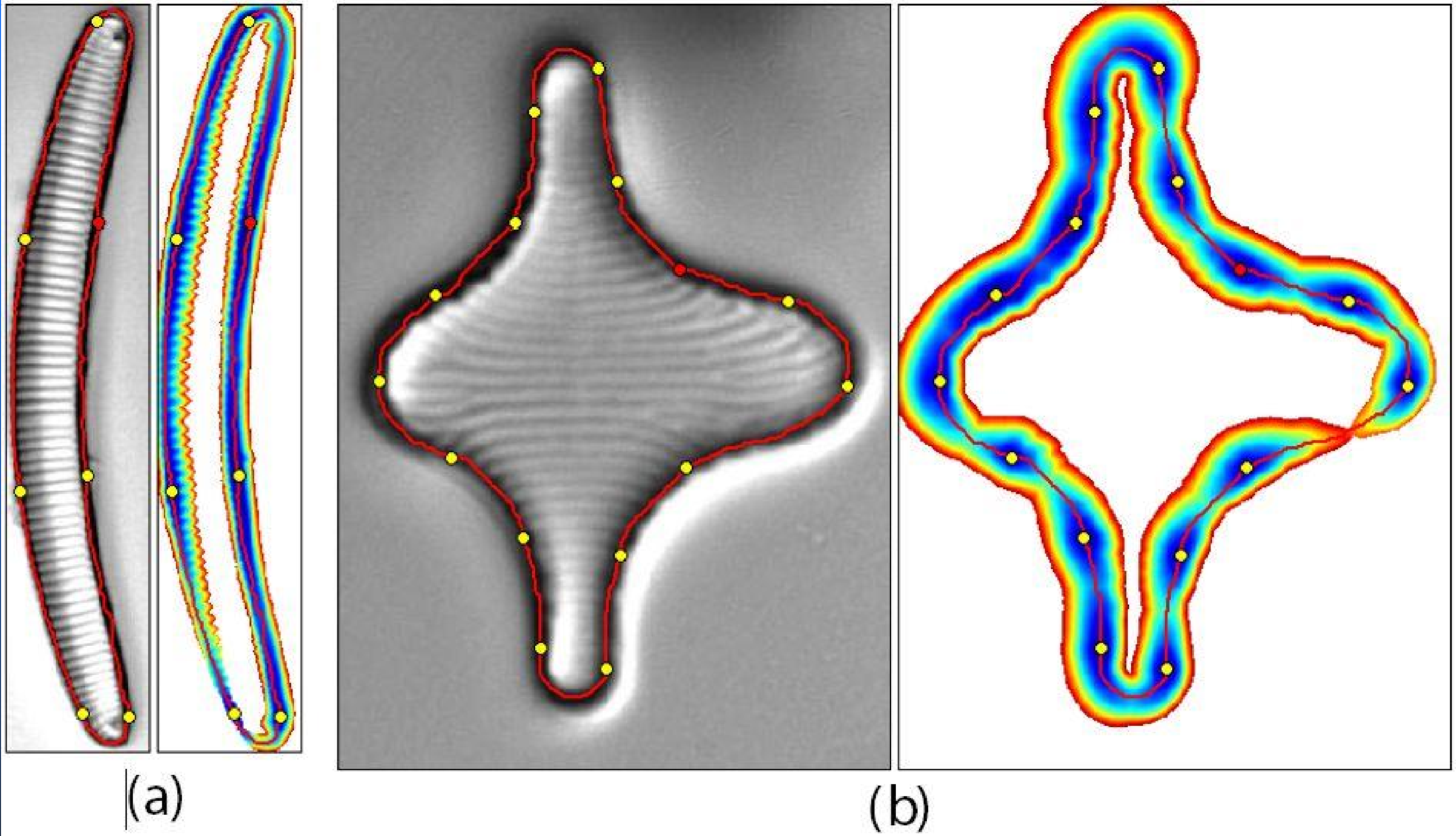


Finding a closed contour by growing minimal paths and adding keypoints



01/02/2017 19:16 Laurent D. COHEN, Huawei 2017 195

Finding a closed contour by growing minimal paths



Keypoints and 3D Minimal Paths for tubular shapes in 2D

(with Li and Yezzi, MICCAIâ09)

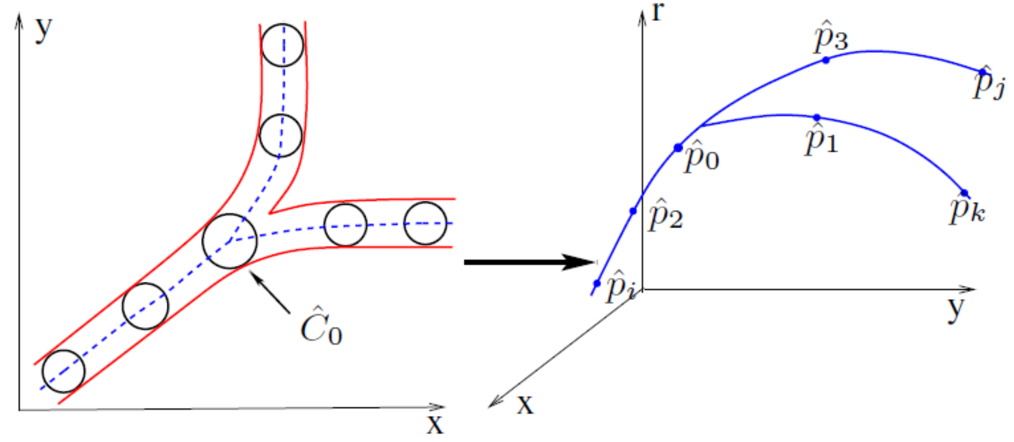
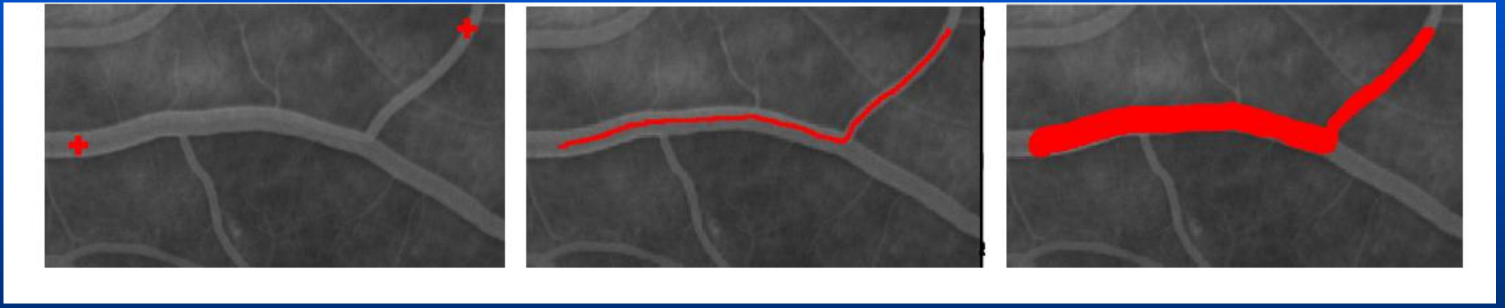


Fig. 1. The entire multi-branch structure extraction is reduced to finding structures between all adjacent key point pairs. The 4D path length D between each key point pair is equal to d_{step}. For easier visualization, the same concept is illustrated here using circles instead of spheres.

01/02/2017 19:16 Laurent D. COHEN, Huawei 2017 228

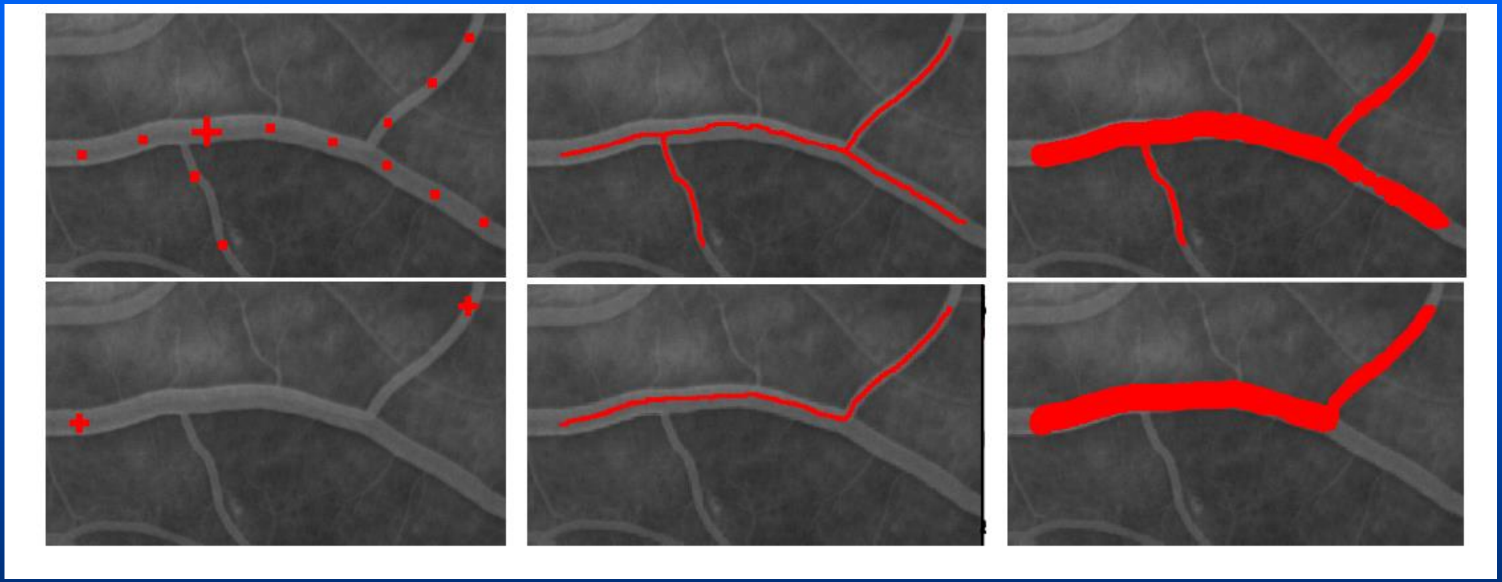
Keypoints and 3D Minimal Paths for tubular shapes in 2D

2D in space , 1D for radius of vessel



01/02/2017 19:16 Laurent D. COHEN, Huawei 2017 229

Keypoints and 3D Minimal Paths for tubular shapes in 2D



01/02/2017 19:16 Laurent D. COHEN, Huawei 2017 230

Keypoints and 3D Minimal Paths for tubular shapes in 2D

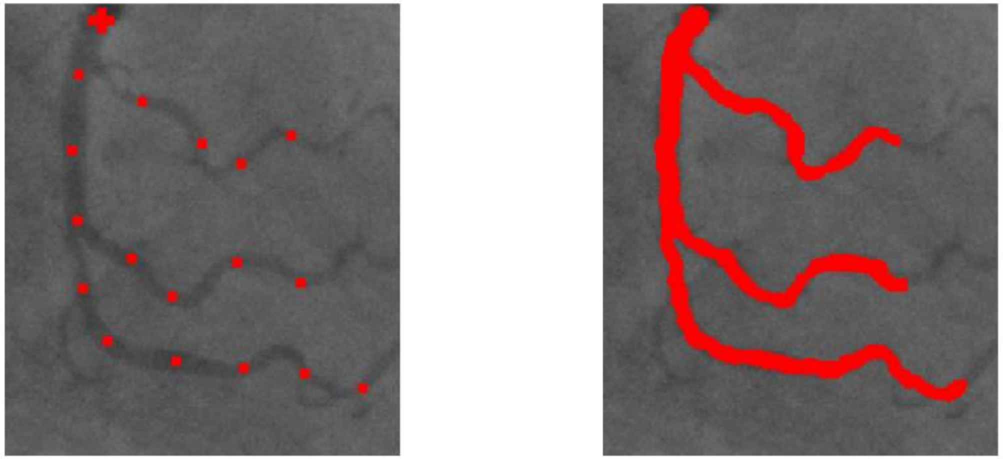
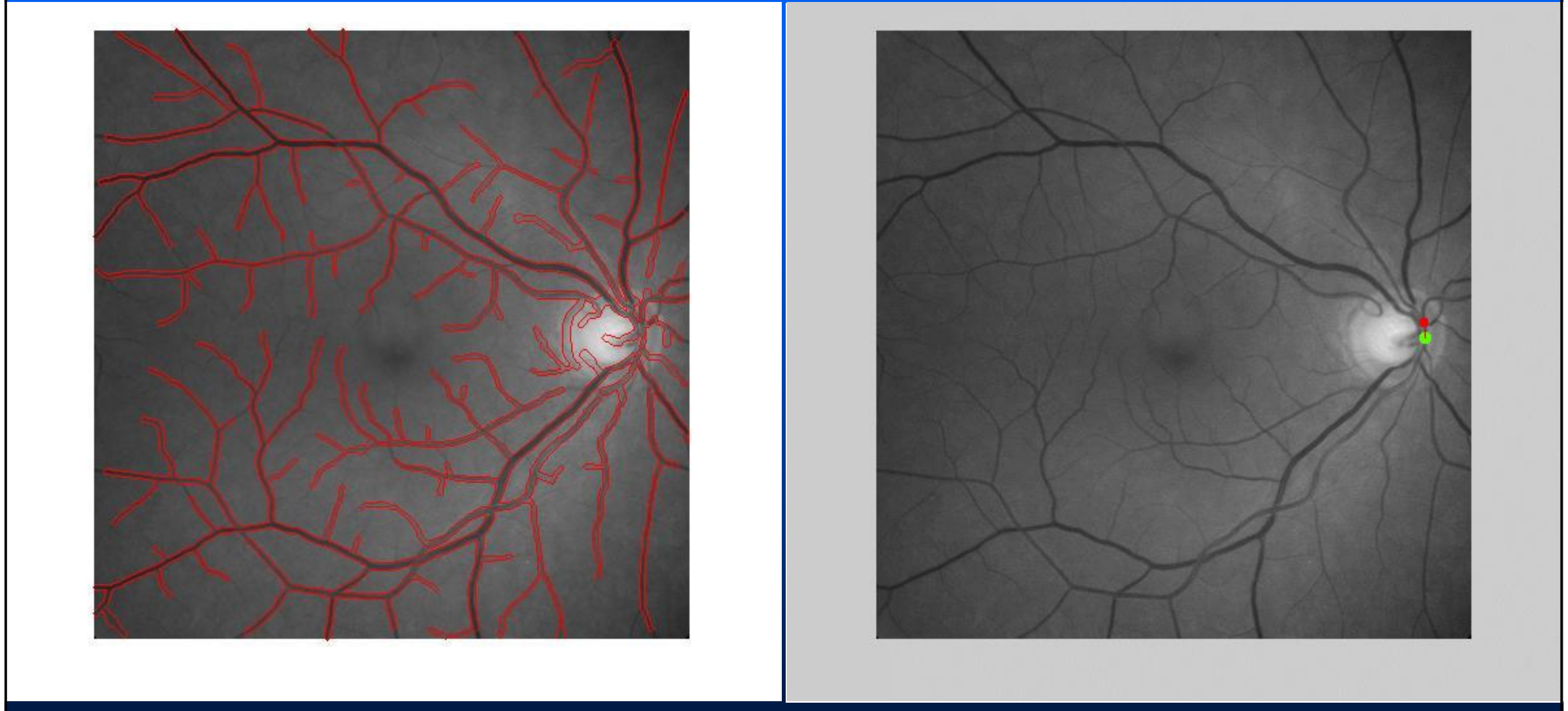


Fig. 3. Segmentation results via the proposed method on another 2D projection angiogram image. Panels from left to right show the initial point and the detected iterative key points and the detected vessel surfaces.

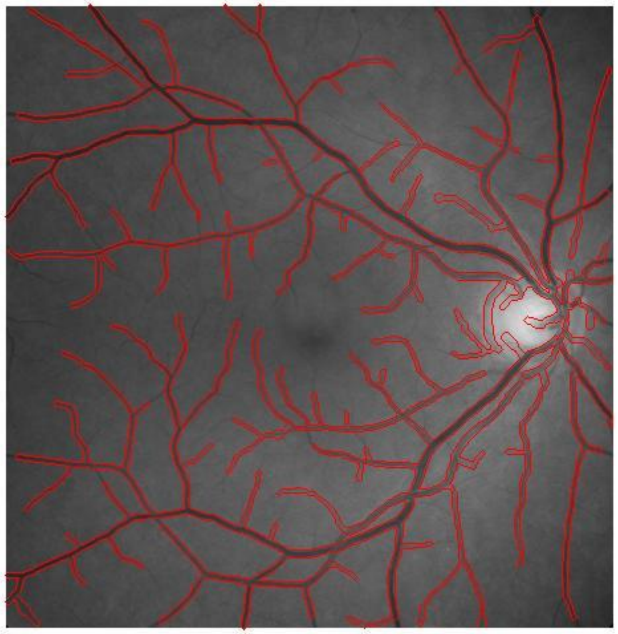
Automatic Keypoint Growing with Mask

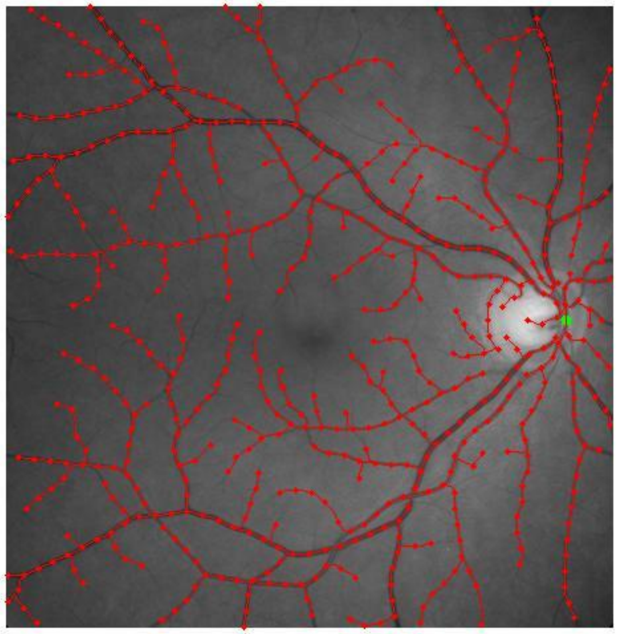
(with Chen Da)



01/02/2017 19:16 Laurent D. COHEN, Huawei 2017 234

Automatic Keypoint Method





Overview

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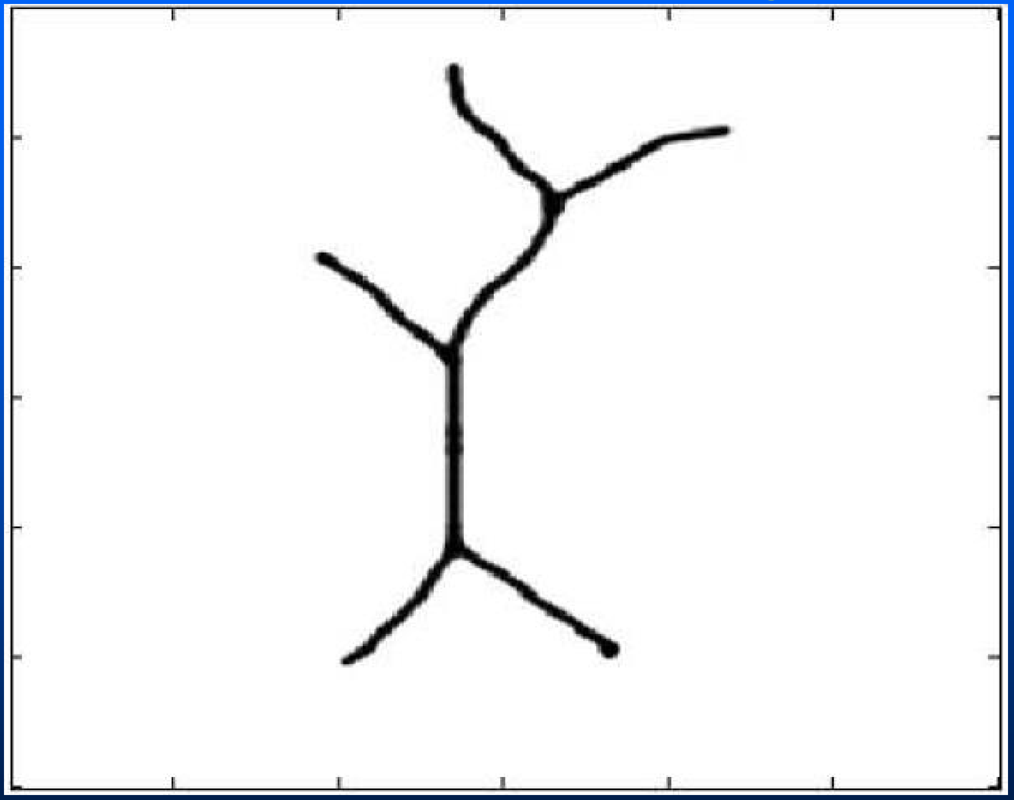
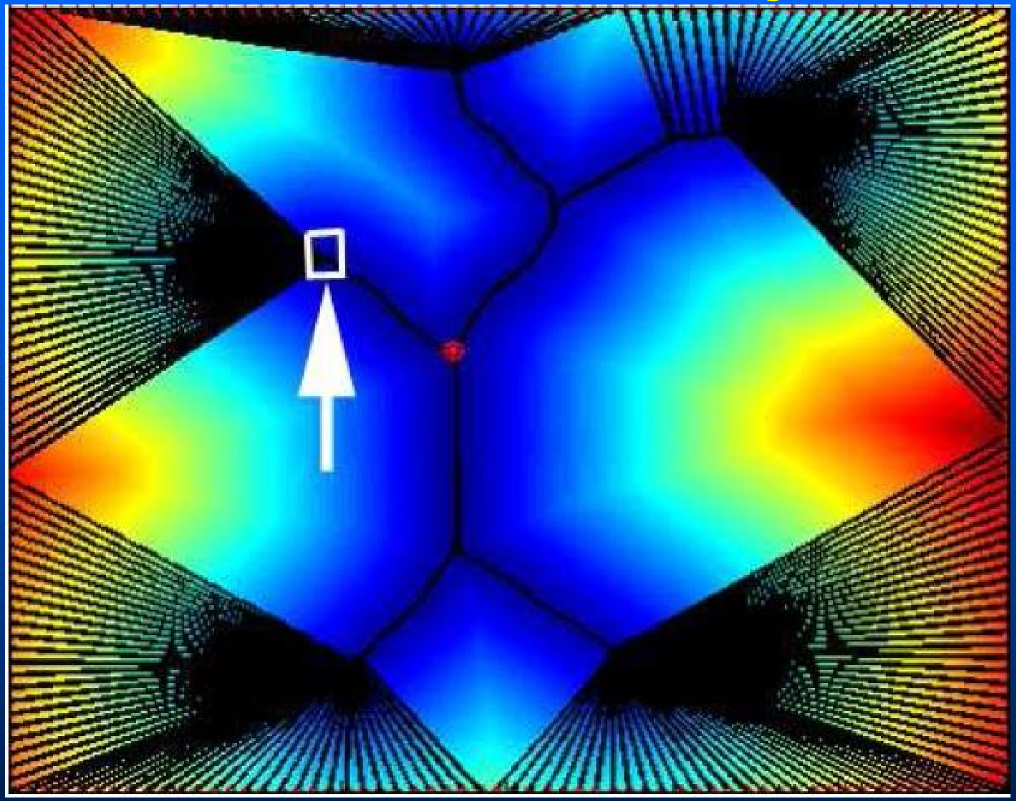
01/02/2017 19:16 Laurent D. COHEN, Huawei 2017 239

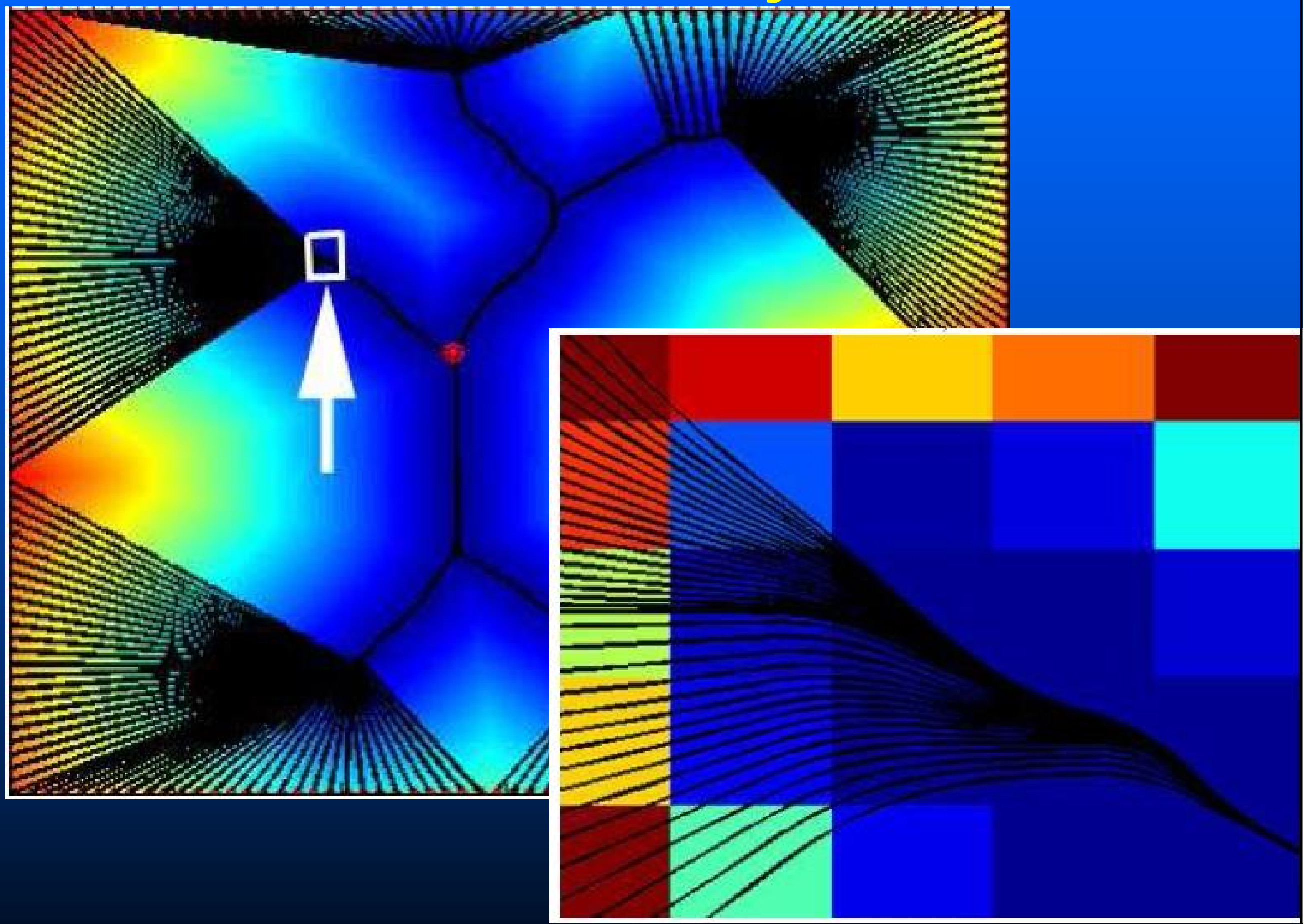
Geodesic Density



Geodesic Density

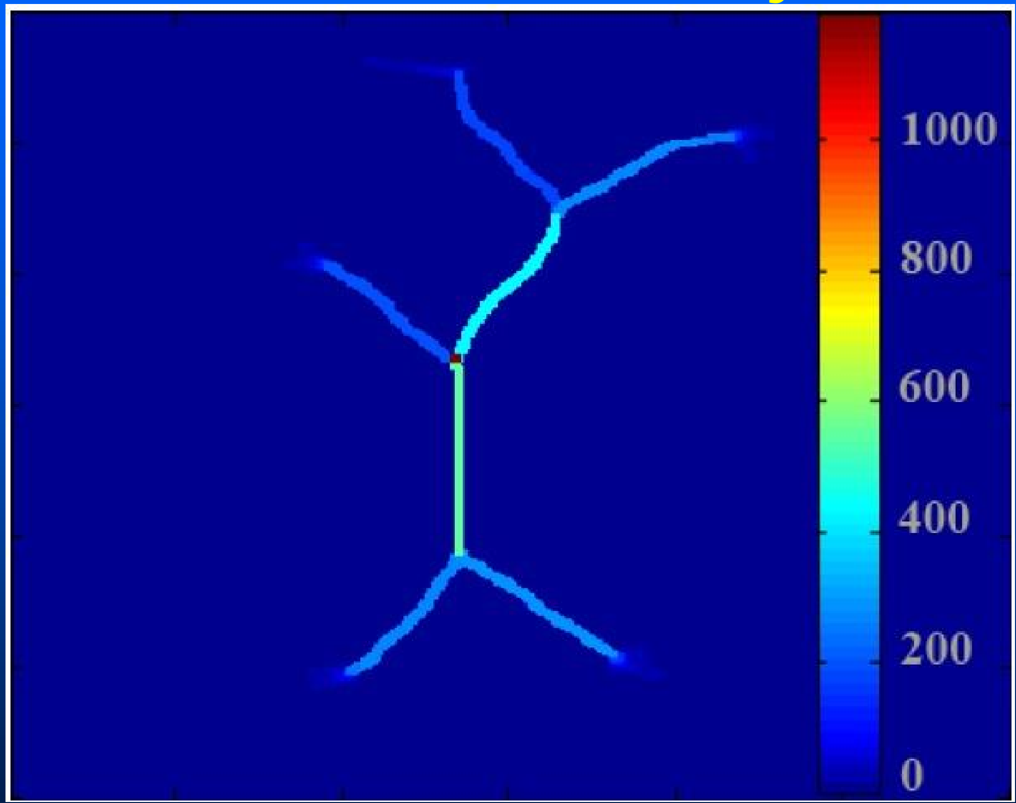


Geodesic Density



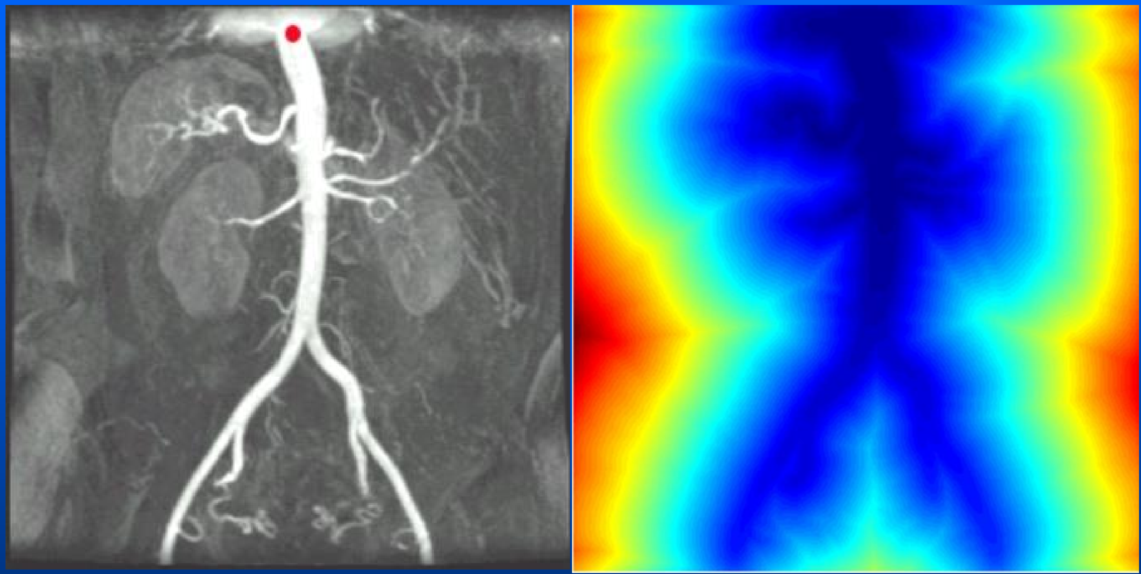
01/02/2017 19:16 Laurent D. COHEN, Huawei 2017 242

Geodesic Density



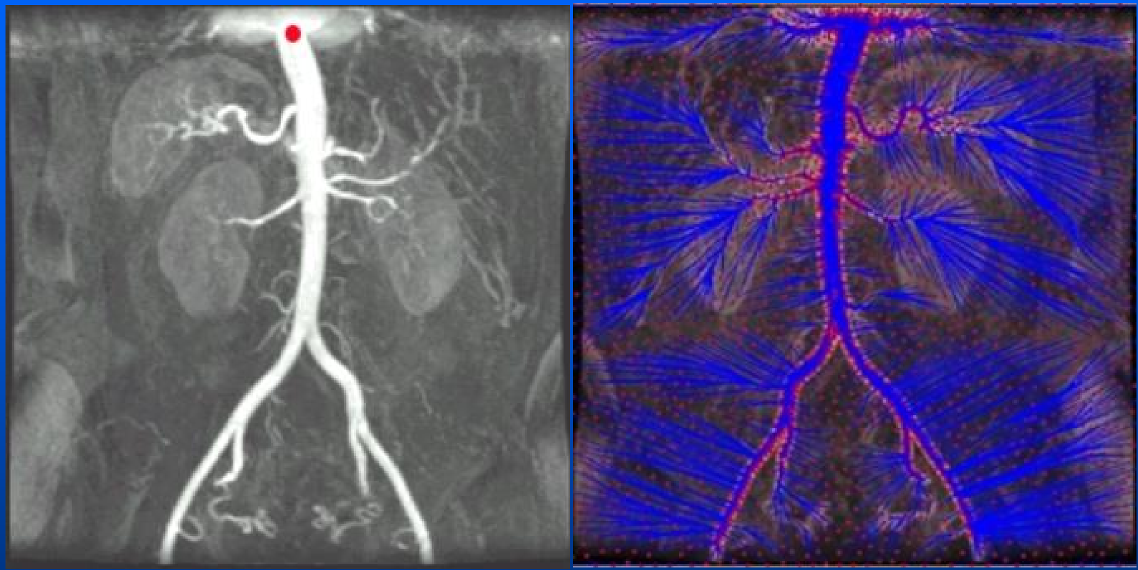
01/02/2017 19:16 Laurent D. COHEN, Huawei 2017 243

Geodesic Density: Real example



01/02/2017 19:16 Laurent D. COHEN, Huawei 2017 257

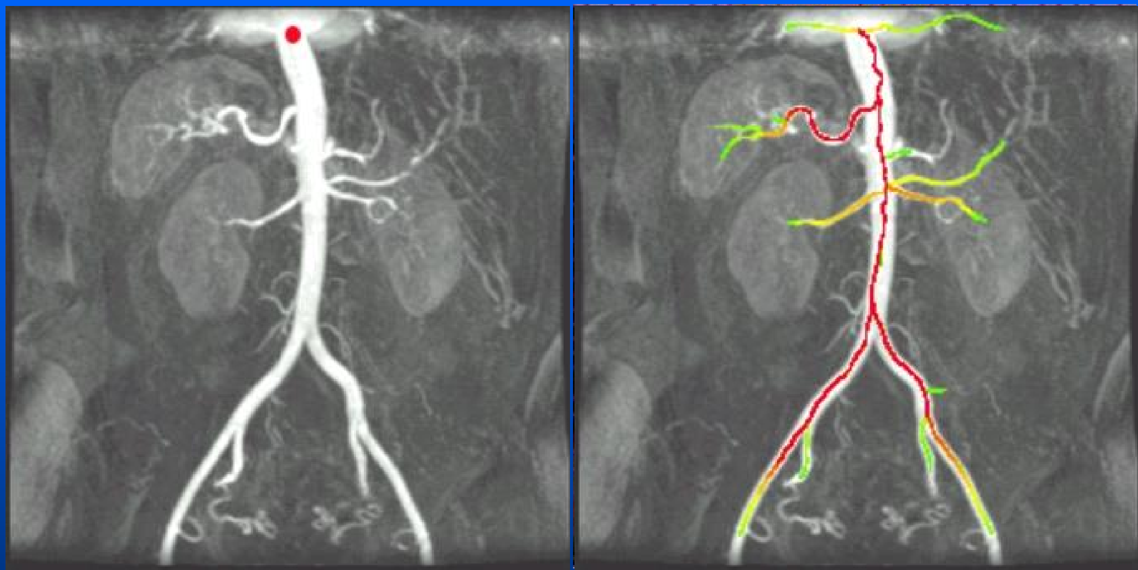
Geodesic Density: adaptive voting



Adaptive voting : 1000 end points

01/02/2017 19:16 Laurent D. COHEN, Huawei 2017 260

Geodesic Density: adaptive voting



Adaptive voting : 1000 end points

01/02/2017 19:16 Laurent D. COHEN, Huawei 2017 261

Conclusion

- Minimally interactive tools for vessels and vascular tree segmentation (tubular branching structures)
- User provides only one initial point and
  sometimes second end point or stopping parameter
- Fast and efficient propagation algorithm
- Models may include orientation and scale of vessels
- Voting approach as a powerful tool to find the structure, which can be completed with other approach.

Thank you !

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