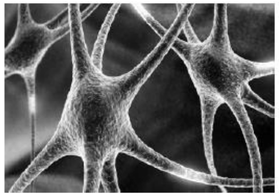
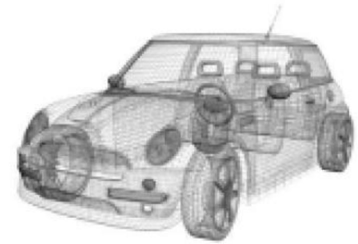
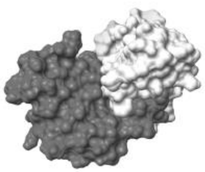
Digital Geometry Processing

Pierre Alliez

Inria Sophia Antipolis

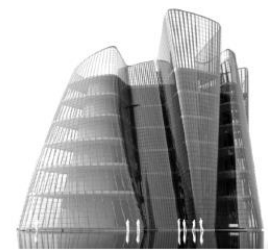






Geometry

γεωμετρία



geo = earth metria = measure

Geometry

γεωμετρία

geo = earth **metria = measure**

microscope

ultrasound

MRI scanner

x-ray diffractometer

stereo camera

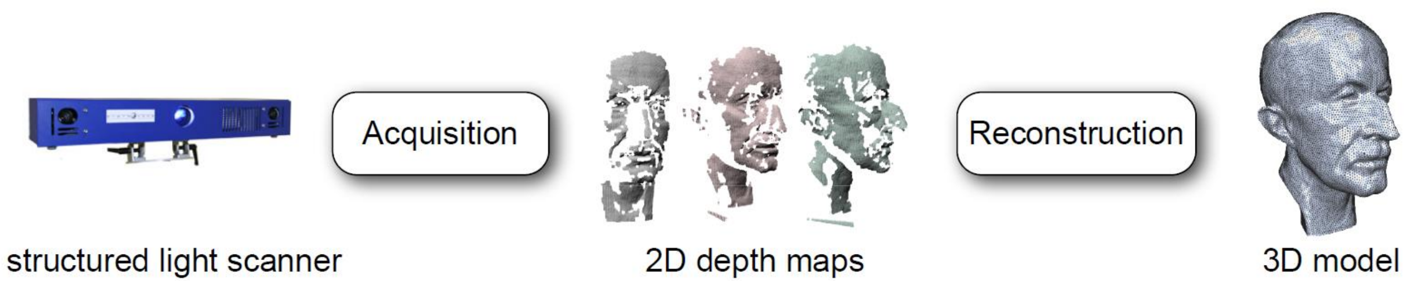
radio telescope

laser scanner

time-of-flight scanner

Digital Geometry

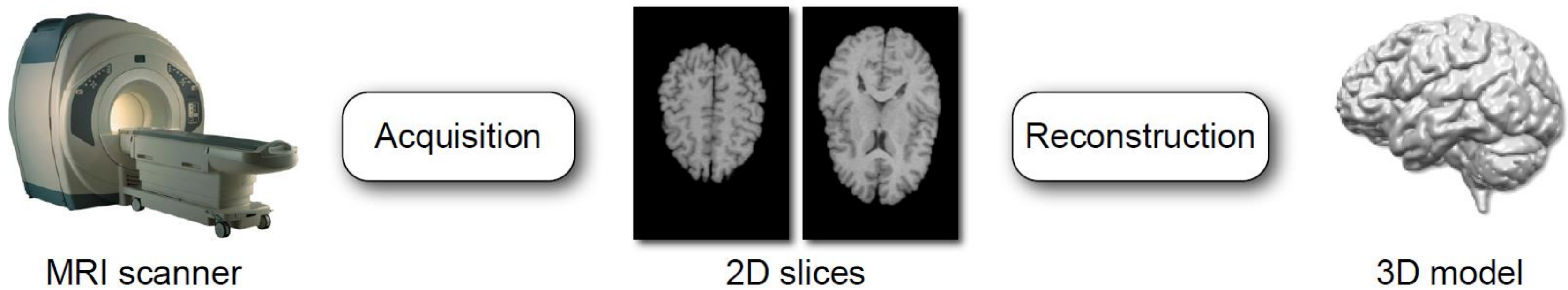
- Entertainment Industry



- Modeling → digital character & set design
- Simulation → computer games, movies, special effects

Digital Geometry

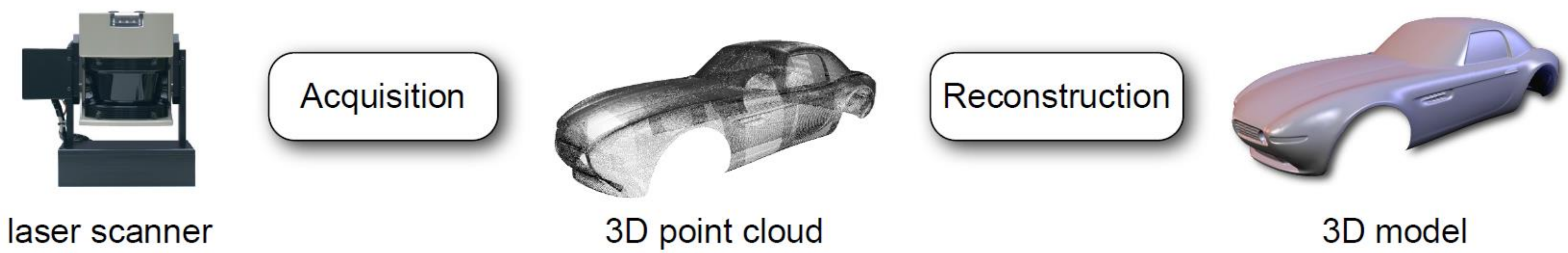
- Medical Applications



- Analysis → diagnosis, operation planning
- Modeling → design of prosthetics
- Simulation → surgery training

Digital Geometry

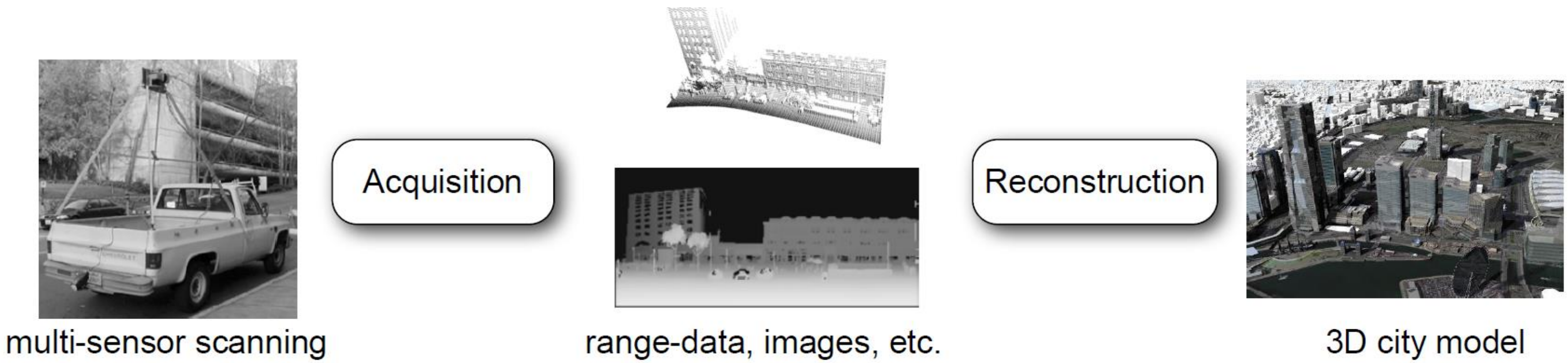
- Engineering Applications



- Analysis → quality control
- Modeling → product design, rapid prototyping
- Simulation → aerodynamics, crash tests

Digital Geometry

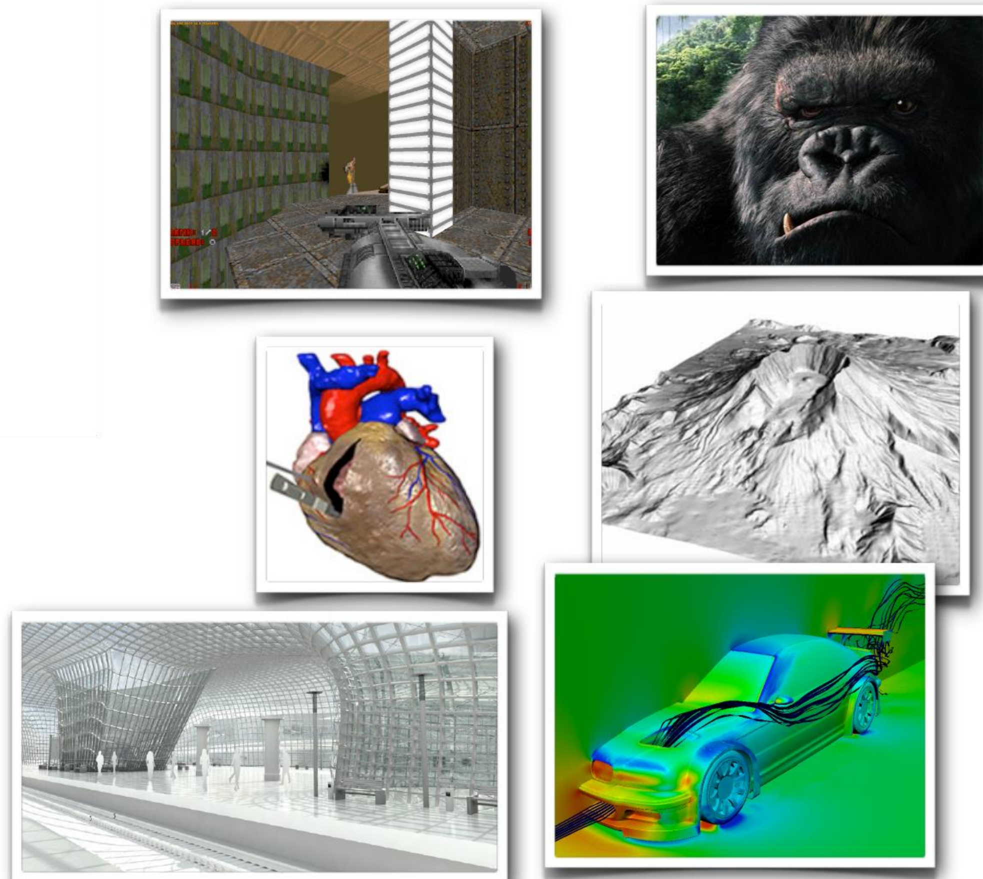
- 3D City Modeling



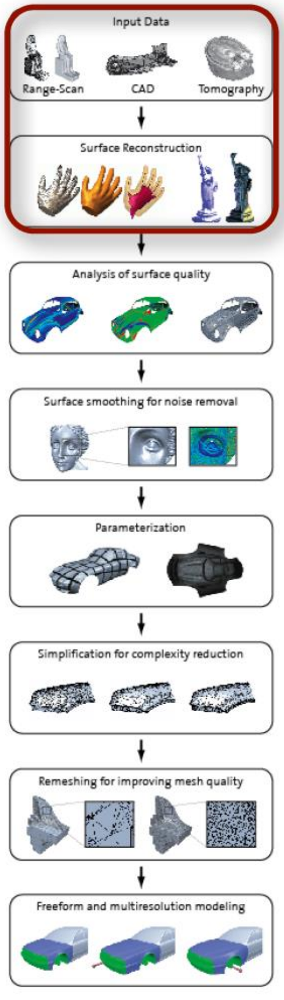
- Analysis → navigation, map design
- Modeling → urban planning, virtual worlds
- Simulation → traffic, pollution, etc.

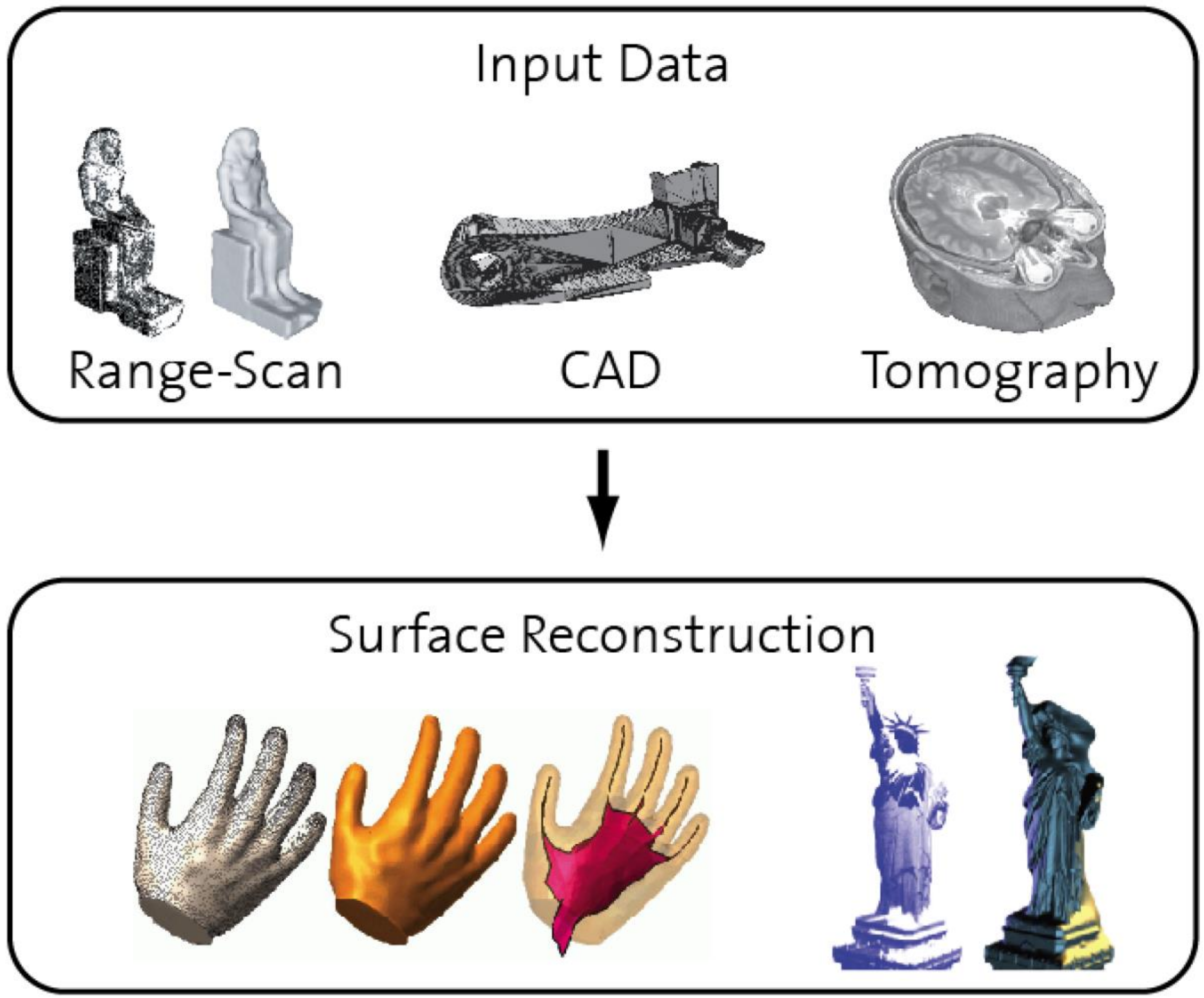
Application Areas

- Computer games
- Movie production
- Engineering
- Cultural Heritage
- Topography
- Architecture
- Medicine
- etc.

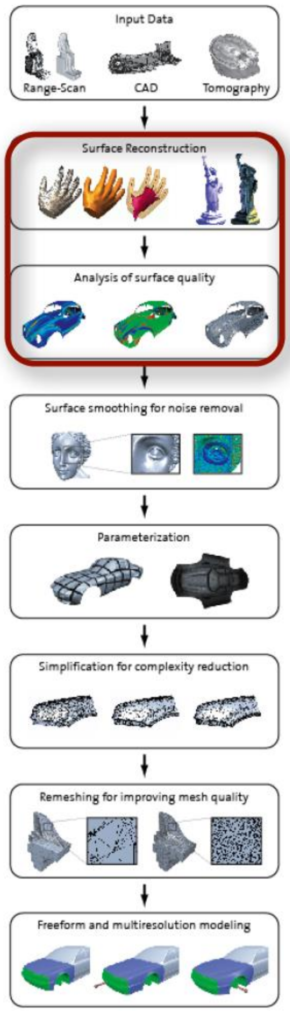


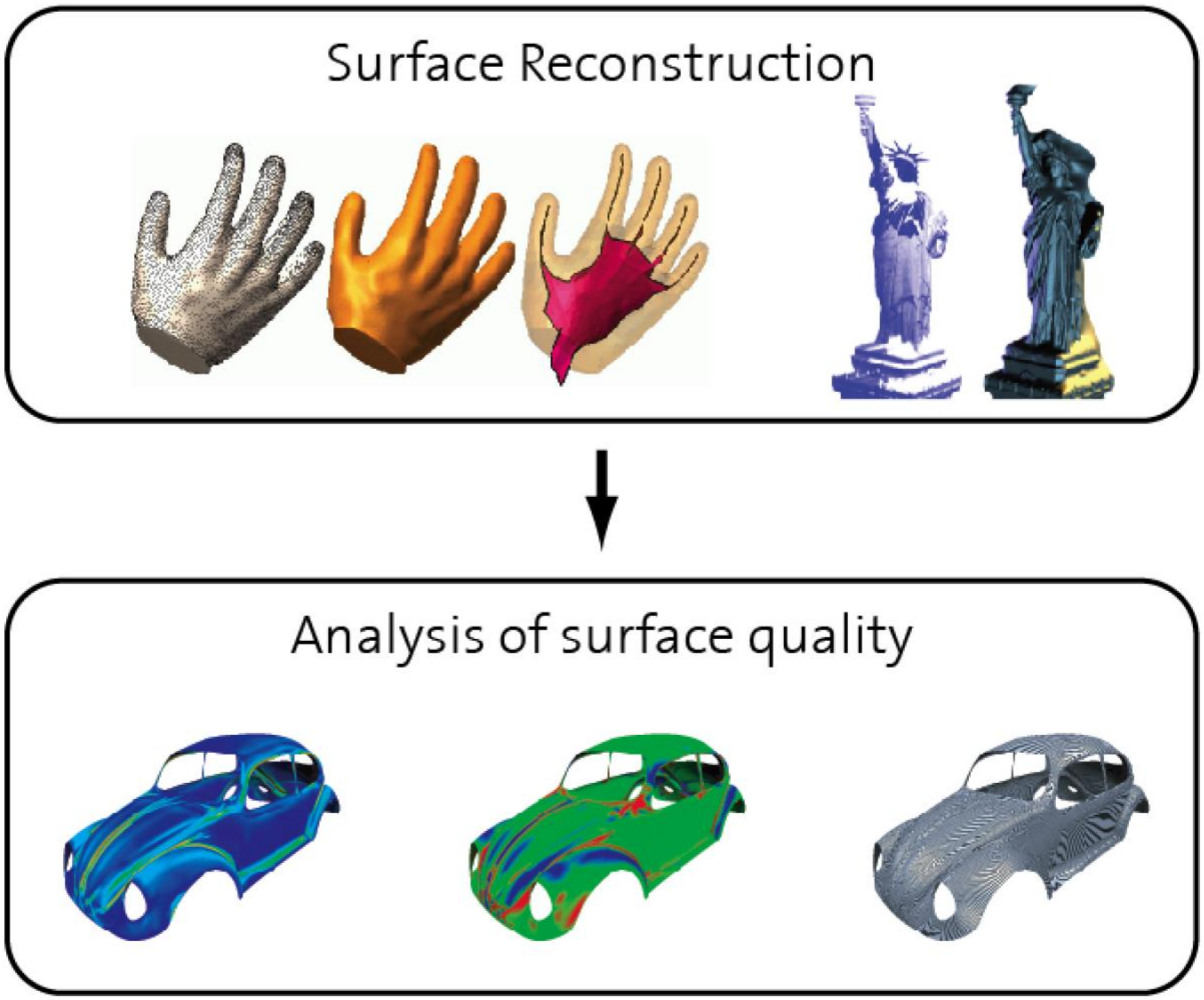
Geometry Processing Pipeline



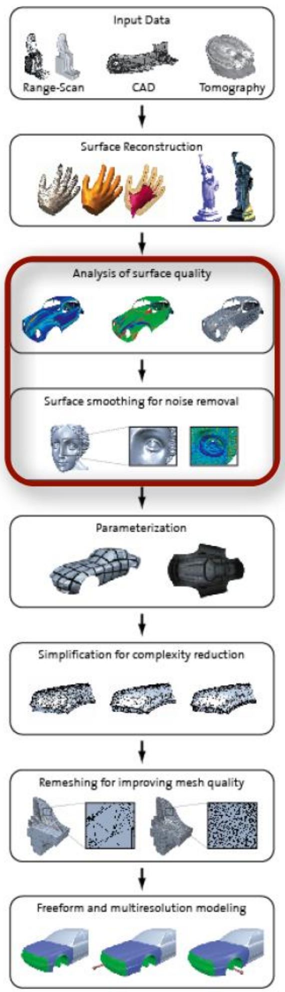


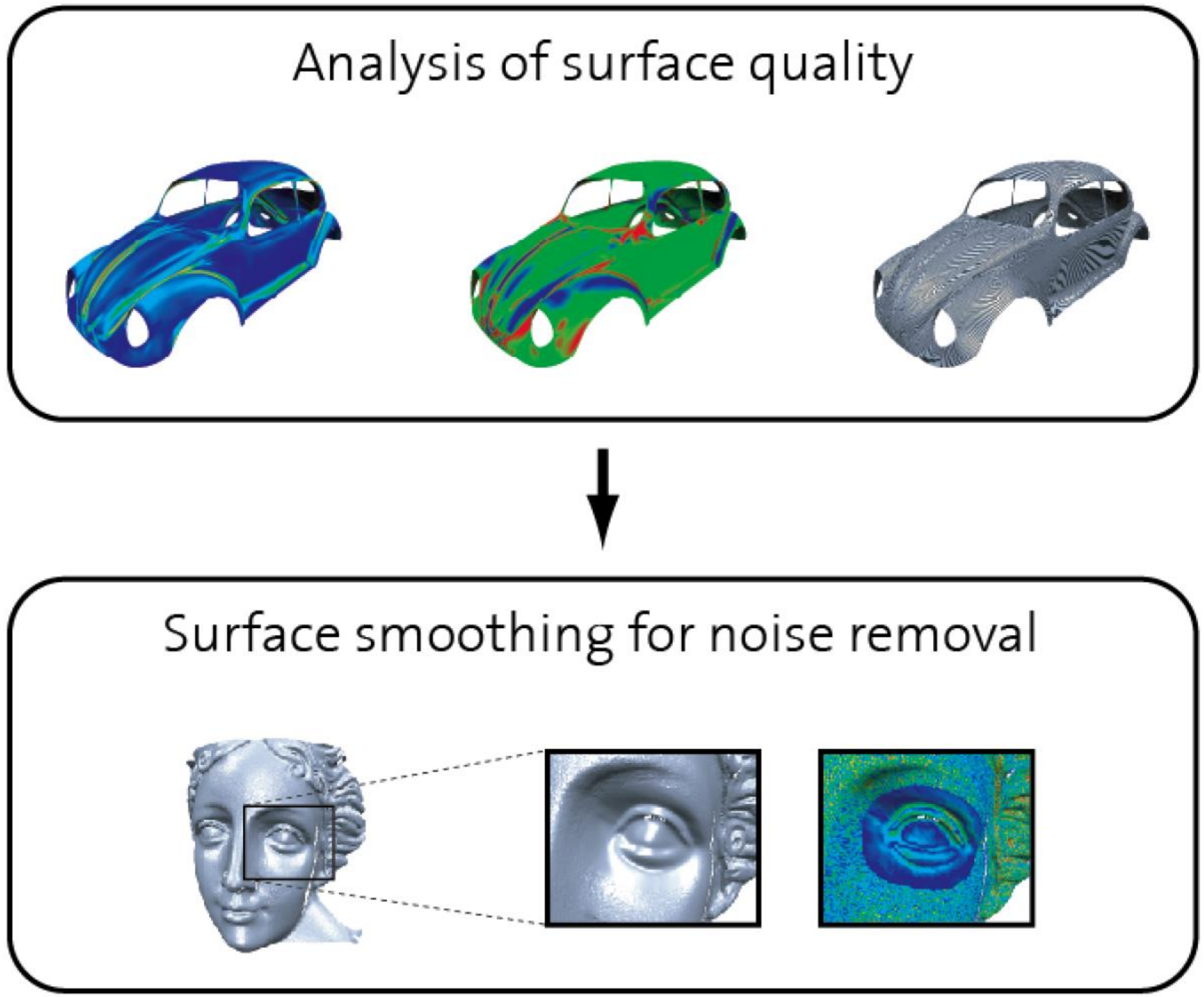
Geometry Processing Pipeline



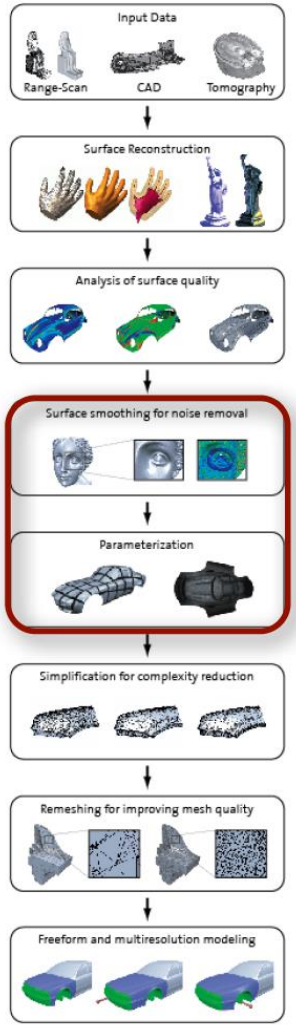


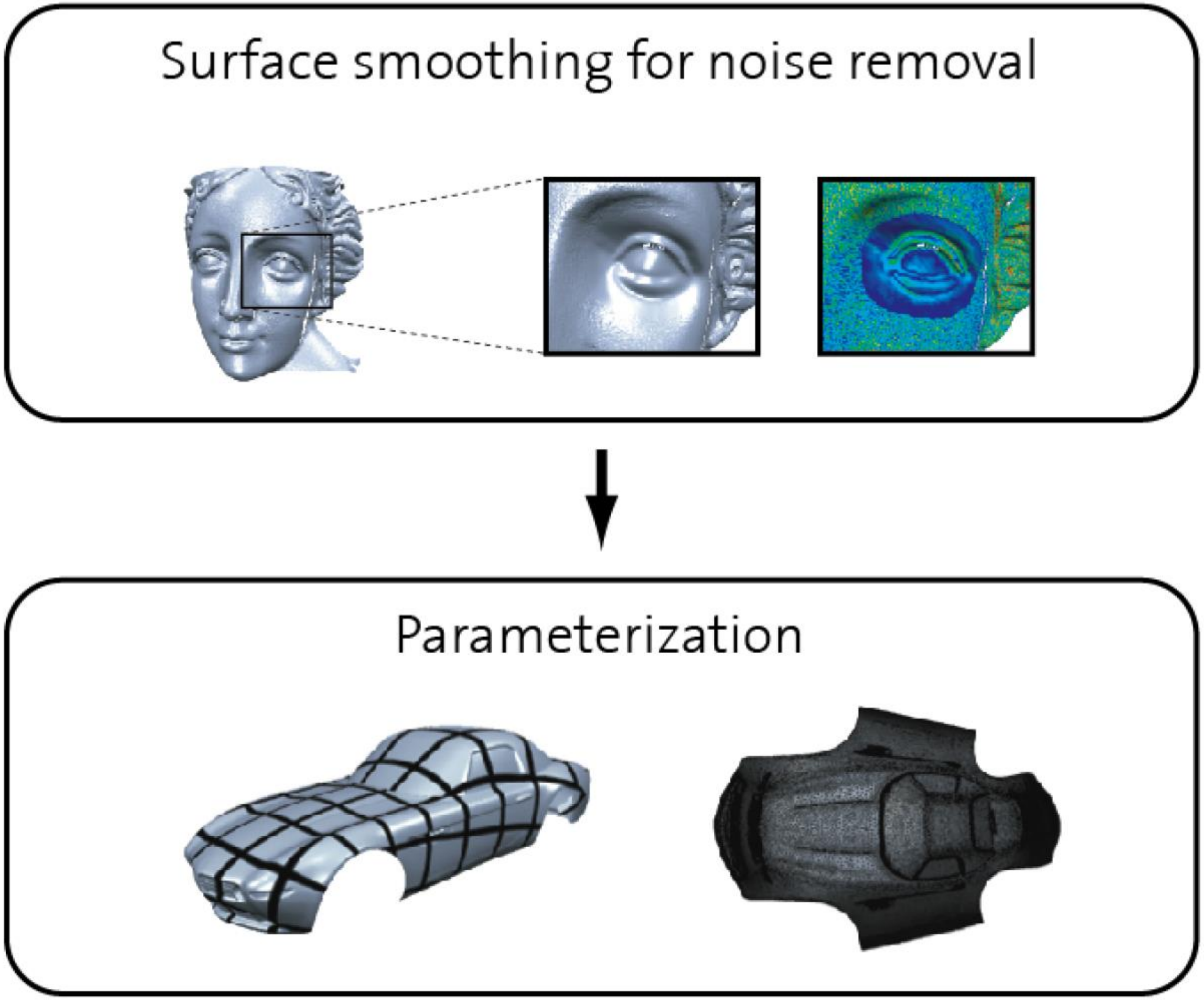
Geometry Processing Pipeline



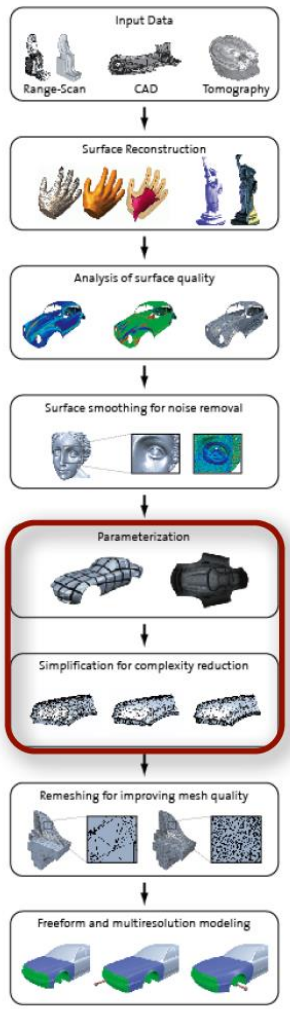


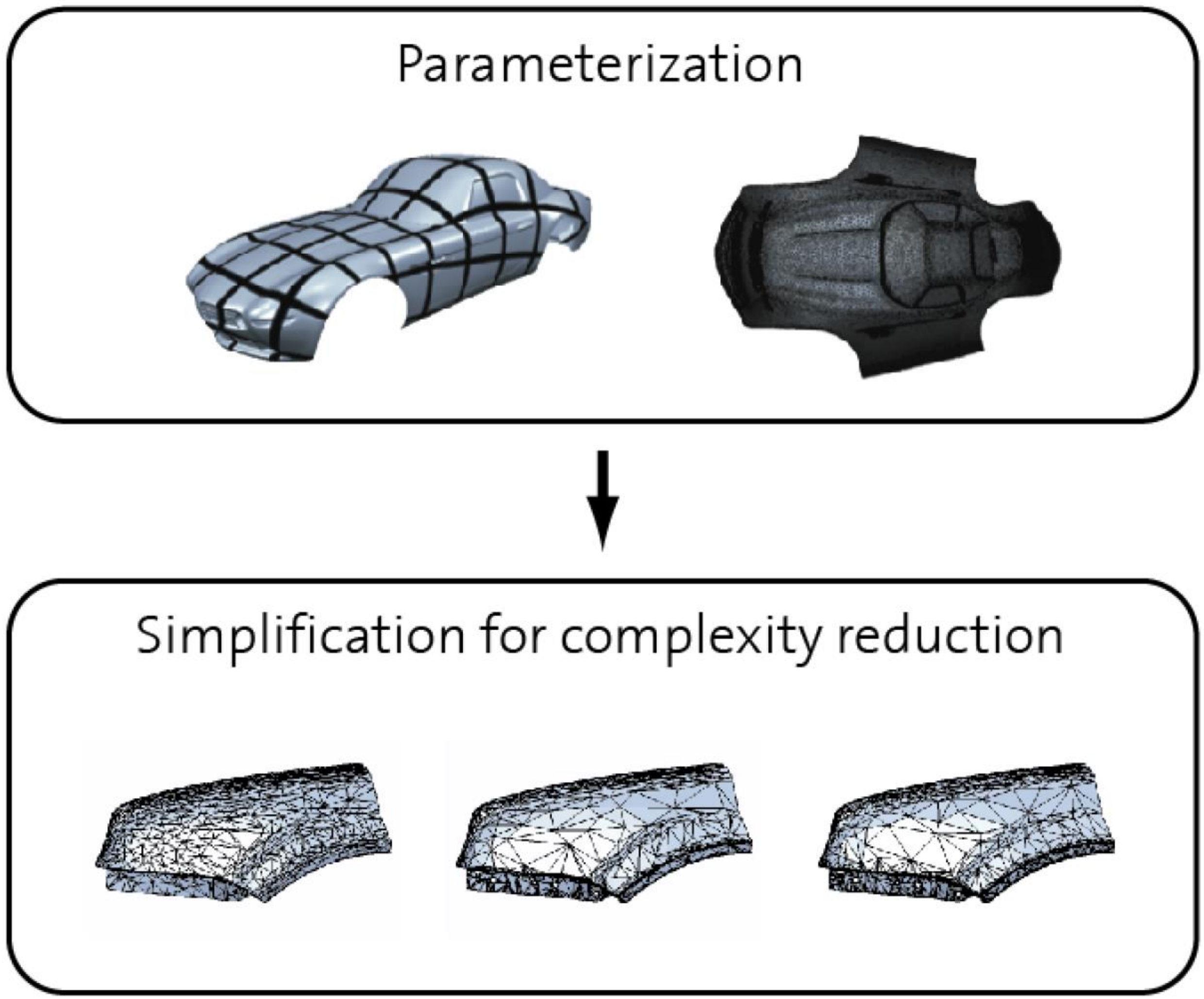
Geometry Processing Pipeline



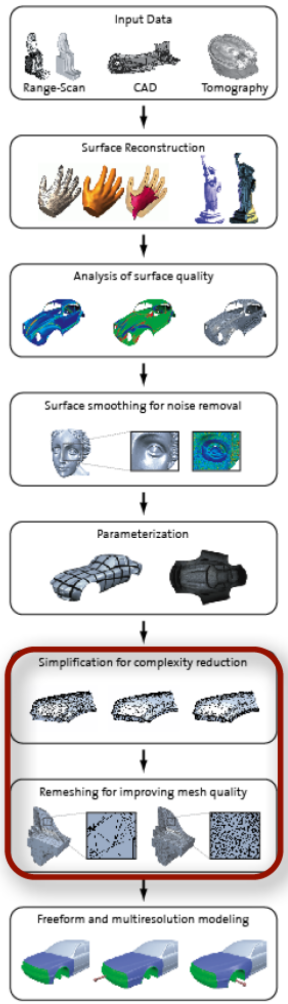


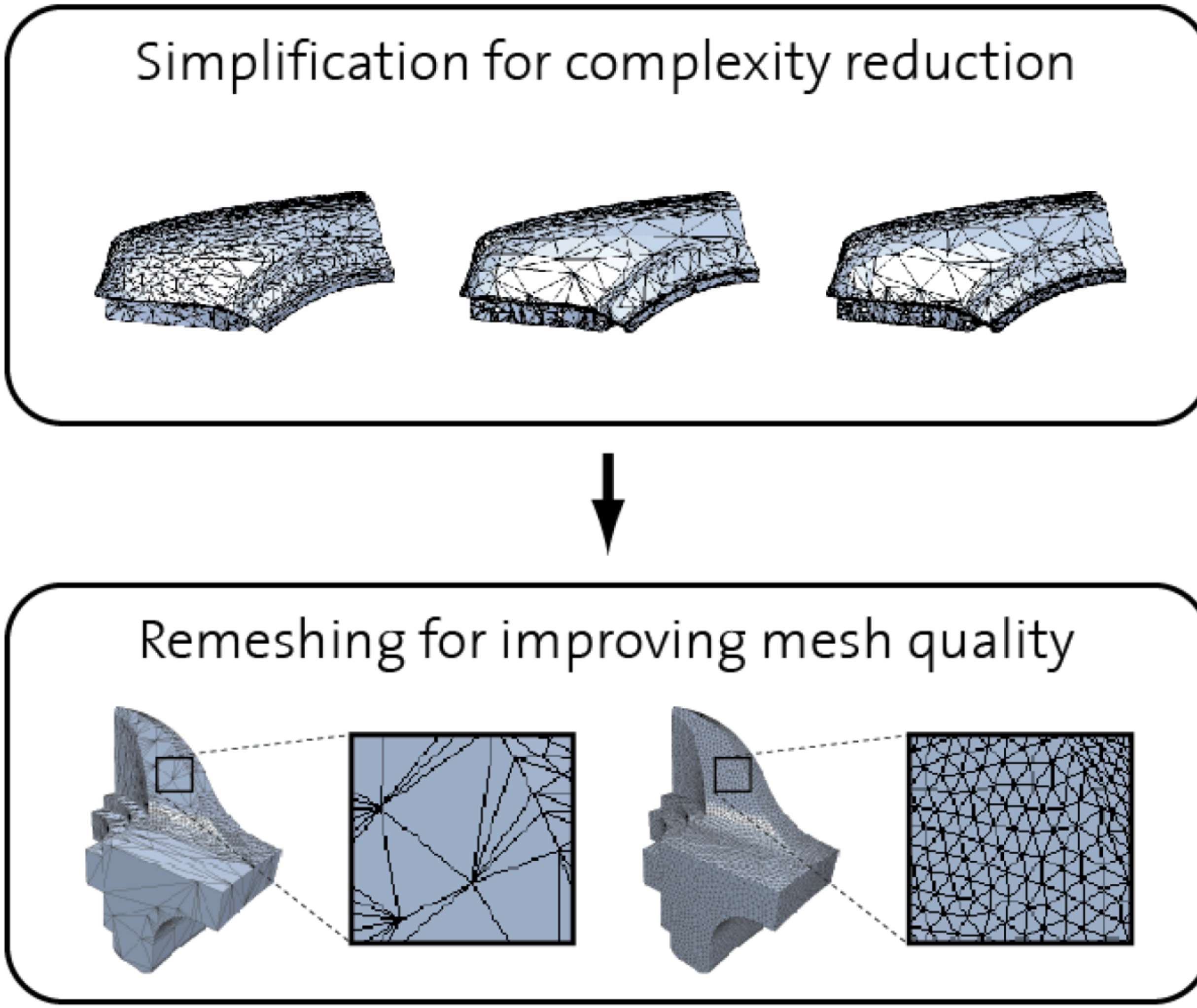
Geometry Processing Pipeline



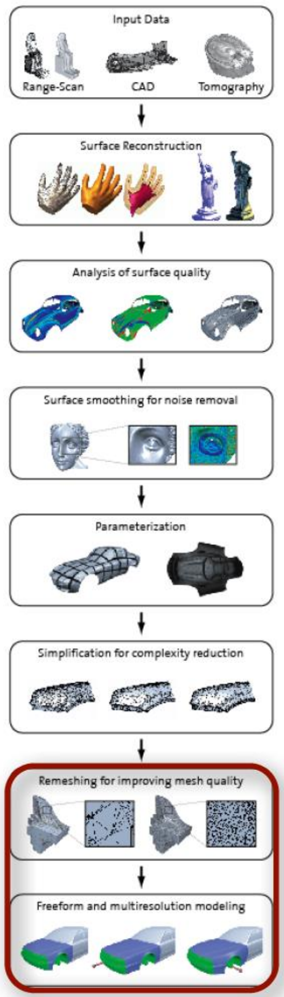


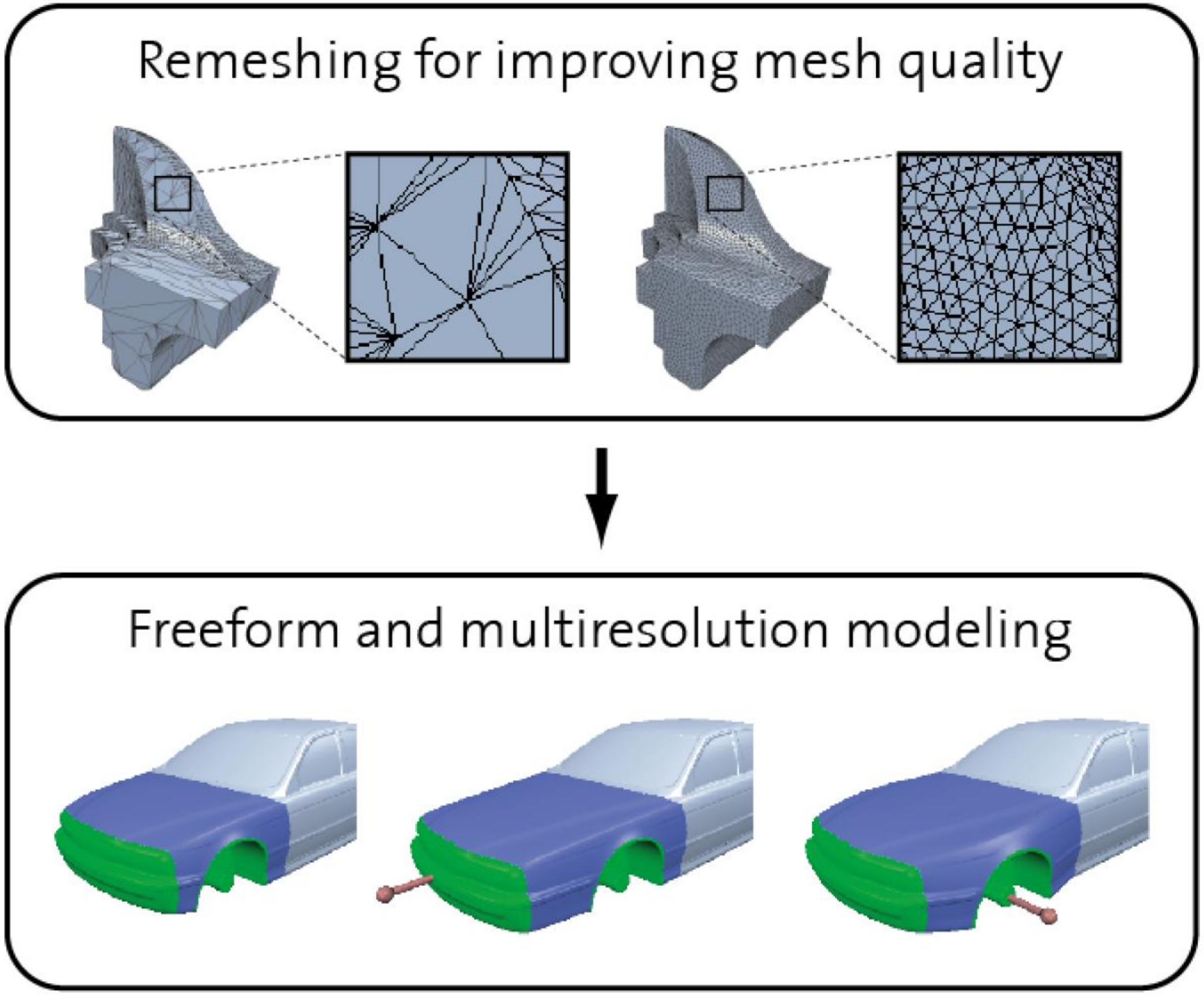
Geometry Processing Pipeline





Geometry Processing Pipeline





Geometry Processing Toolbox

- Geometric Modeling
  - Methods & algorithms for representing and processing geometric objects

- Geometry processing
  - Core algorithms?
  - Efficient implementations?

Shape Reconstruction

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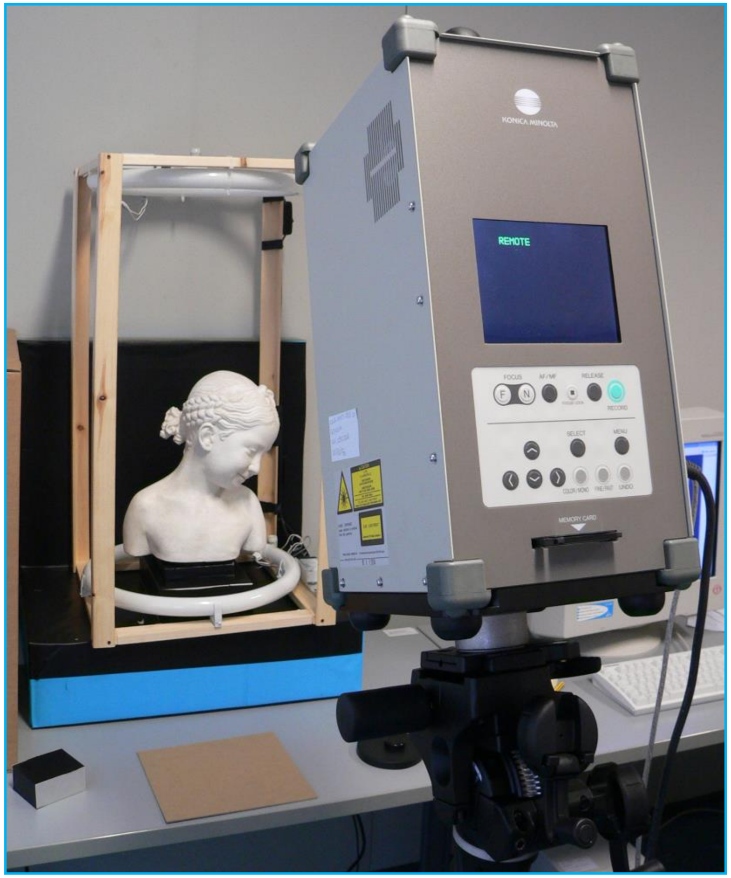
Inria

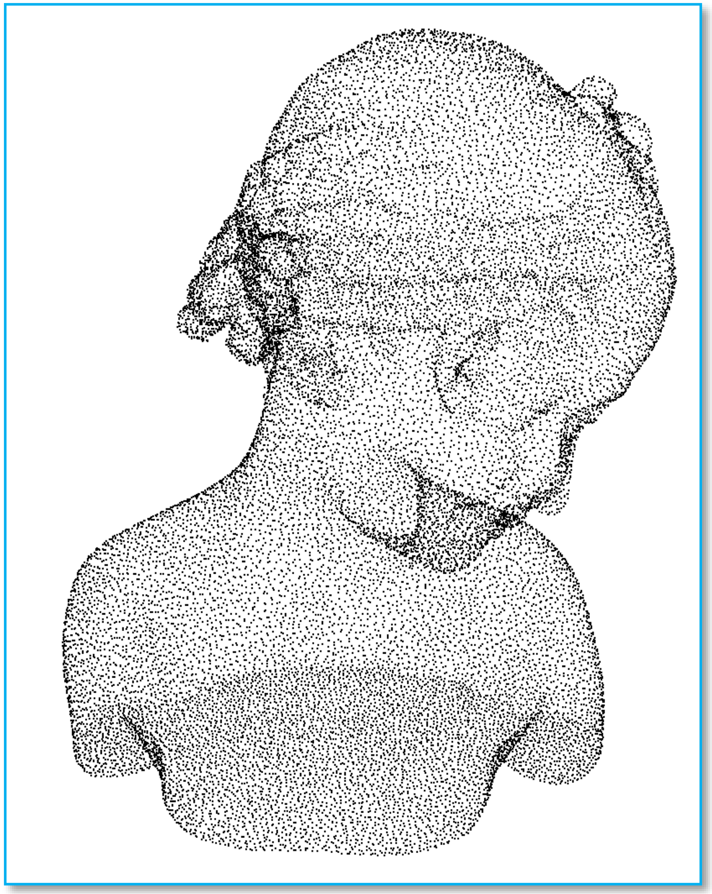
Outline

- Sensors
- Problem statement
- Computational Geometry
  - Voronoi/Delaunay
  - Alpha-shapes
  - Crust
- Variational formulations
  - Poisson reconstruction

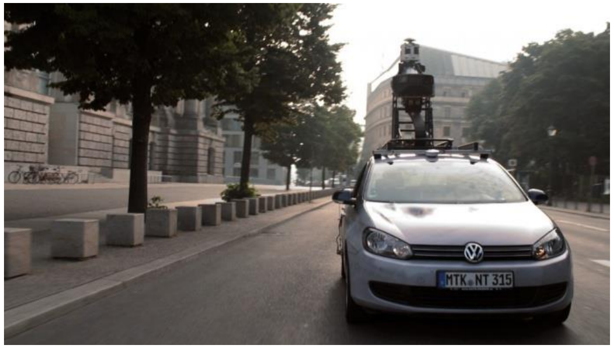
SENSORS

Laser scanning

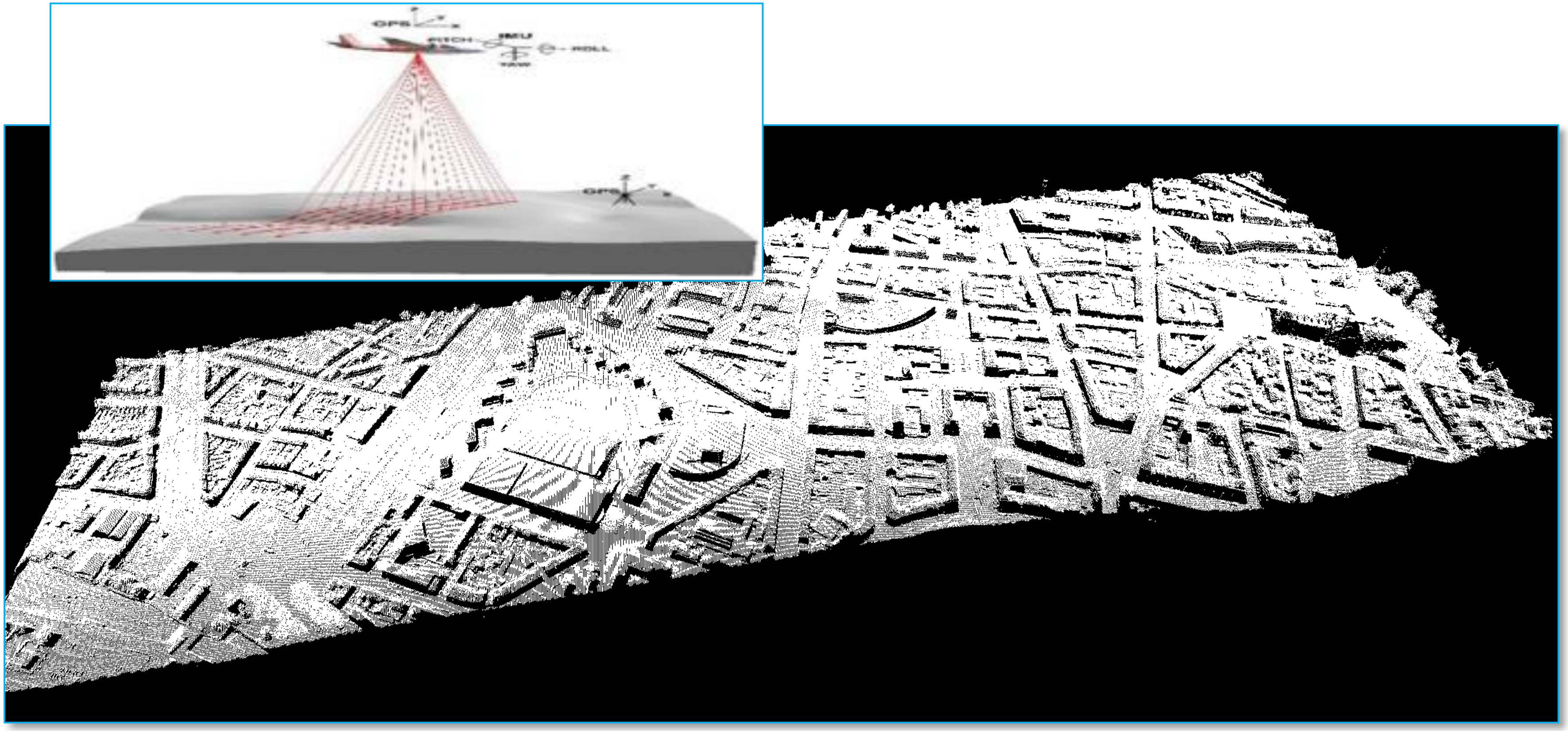




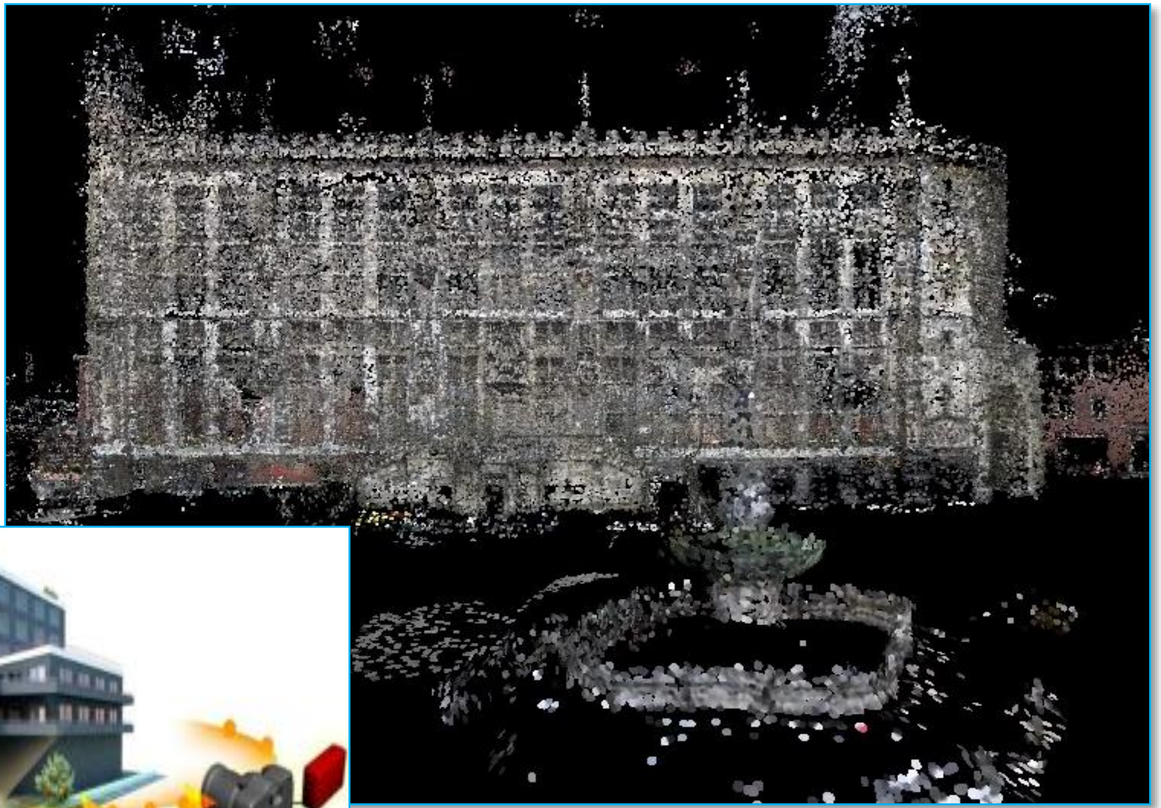
Car-based Laser



Airborne Lidar



Multi-View Stereo (MVS)



Depth Sensors

PROBLEM STATEMENT

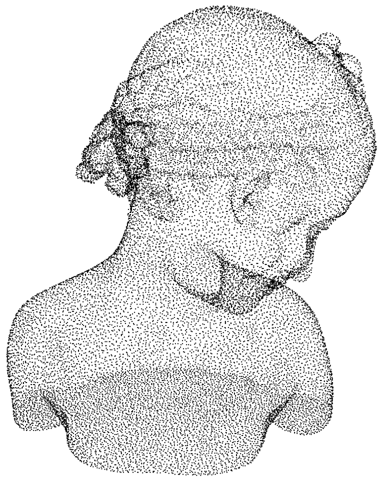
Reconstruction Problem

Input: point set P sampled over a surface S:

Non-uniform sampling

With holes

With uncertainty (noise)



point set

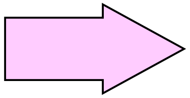
Output: surface

Approximation of S in terms of topology and geometry

Desired:

Watertight

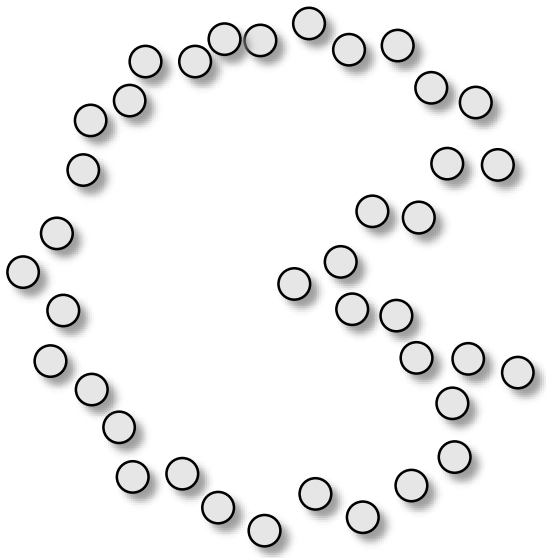
Intersection free



reconstruction

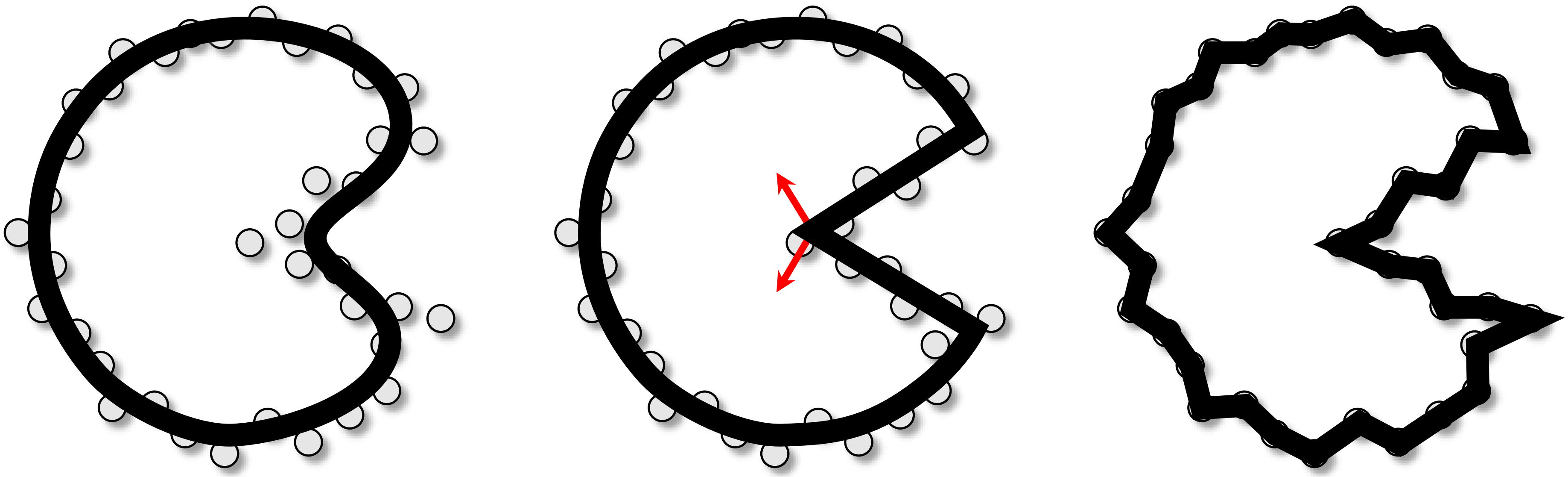
surface

Ill-posed Problem



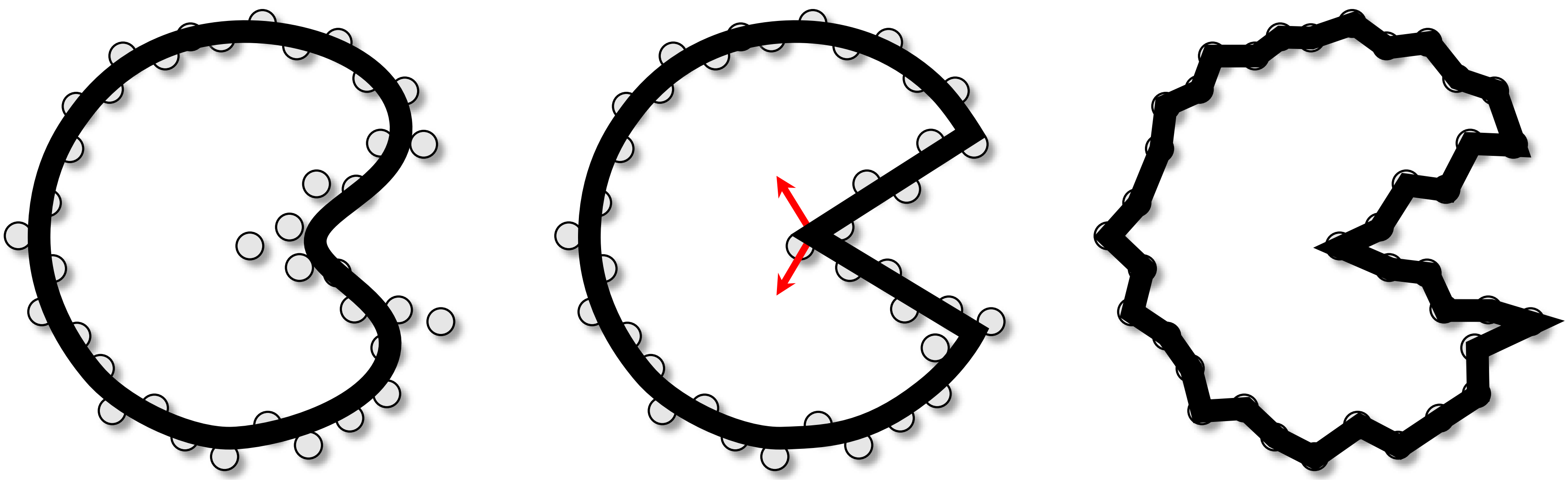
Many candidate surfaces for the reconstruction problem!

Ill-posed Problem



Many candidate surfaces for the reconstruction problem! How to pick?

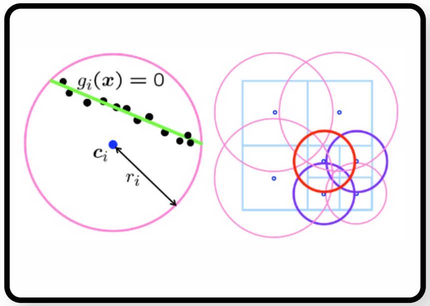
Priors



Smooth | Piecewise Smooth | "Simple"

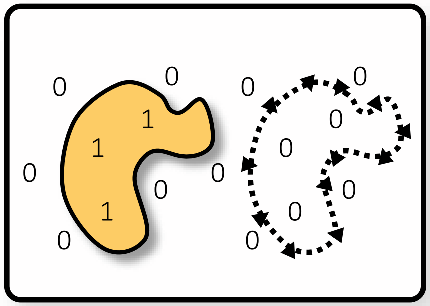
Surface Smoothness Priors

Local Smoothness



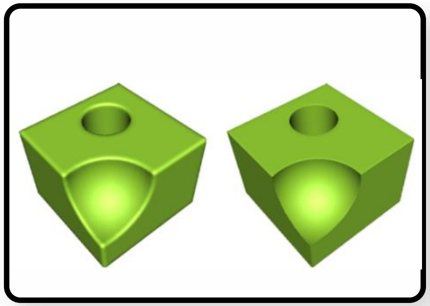
Local fitting
No control away from data
Solution by interpolation

Global Smoothness



Global: linear, eigen, graph cut, ...
Robustness to missing data

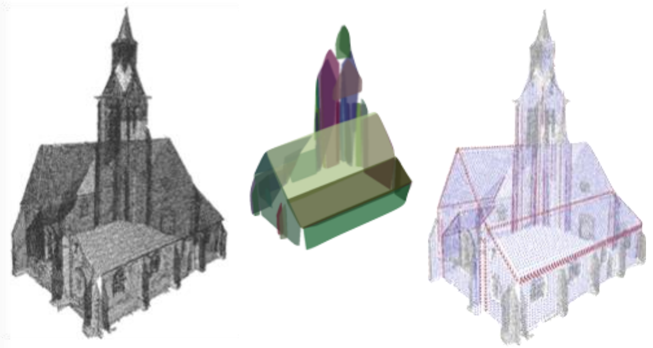
Piecewise Smoothness



Sharp near features
Smooth away from features

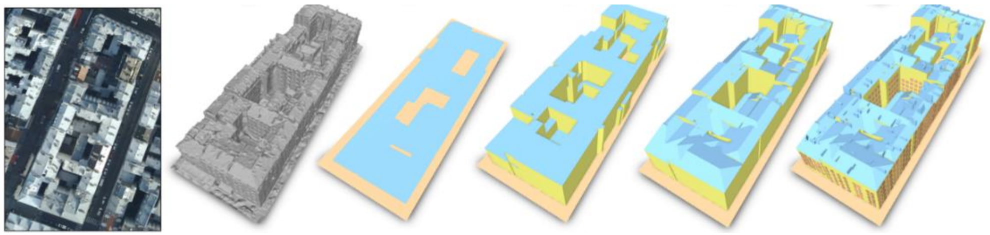
Domain-Specific Priors

Surface Reconstruction
by Point Set Structuring



[Lafarge - A. EUROGRAPHICS 2013]

LOD Reconstruction
for Urban Scenes



[Verdie, Lafarge - A. ACM Transactions on Graphics 2015]

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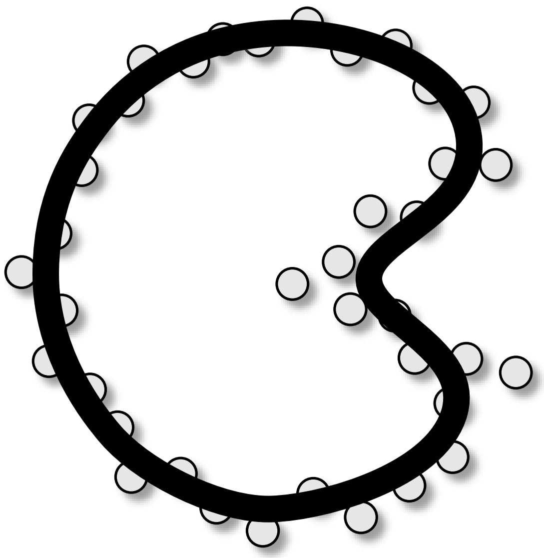
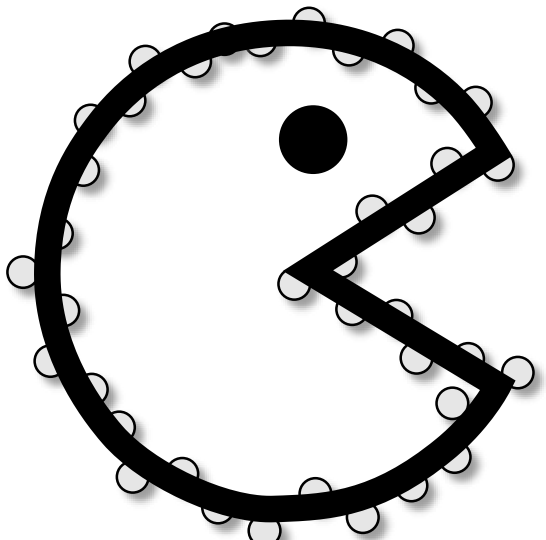
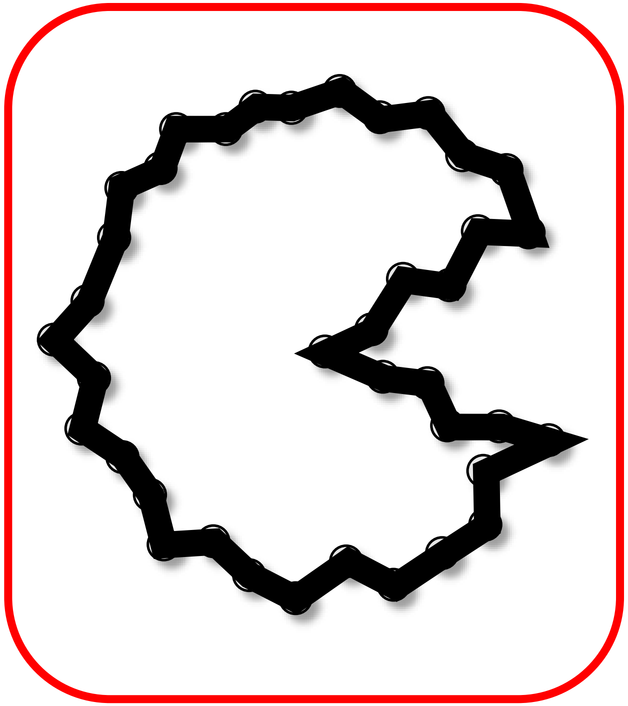
Warm-up



Smooth



Piecewise Smooth



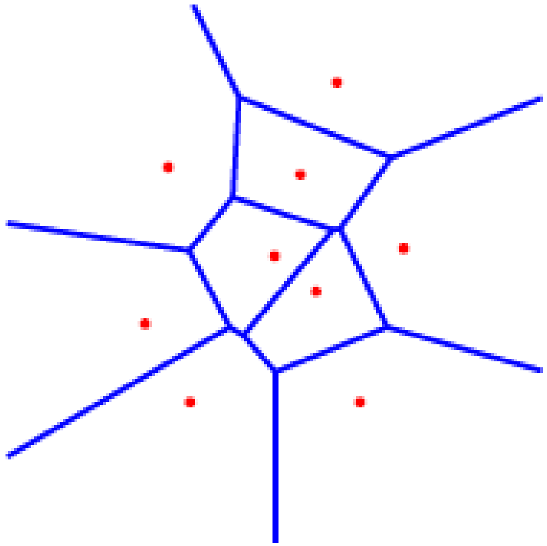
“Simple”

VORONOI / DELAUNAY

Voronoi Diagram

Let $\mathcal{E} = \{\mathbf{p_1}, \ldots, \mathbf{p_n}\}$ be a set of points (so-called sites) in $\mathbb{R}^d$. We associate to each site $\mathbf{p_i}$ its Voronoi region $V(\mathbf{p_i})$ such that:

$$V(\mathbf{p_i}) = \{\mathbf{x} \in \mathbb{R}^d : \|\mathbf{x} - \mathbf{p_i}\| \leq \|\mathbf{x} - \mathbf{p_j}\|, \forall j \leq n\}.$$

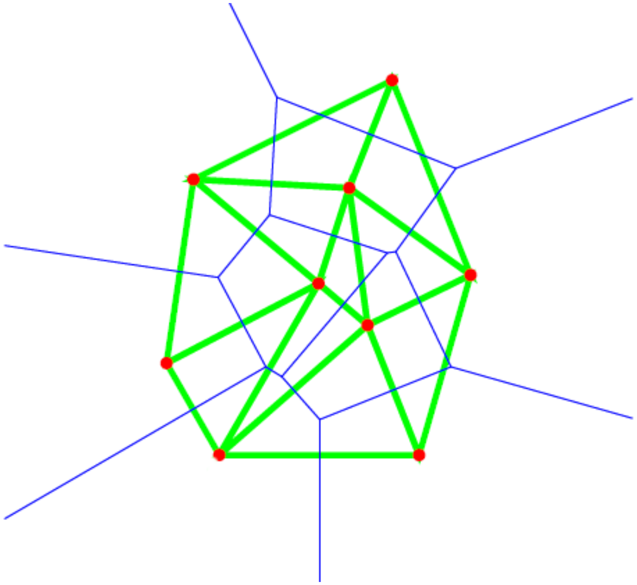


http://www.cgal.org

Delaunay Triangulation

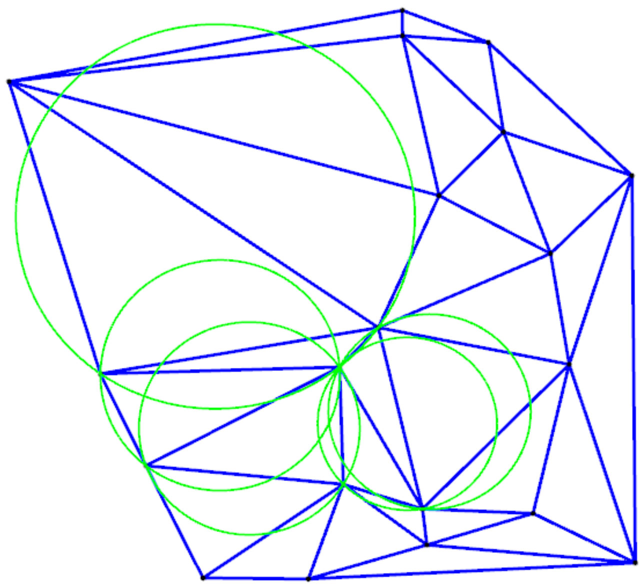
Dual structure of the Voronoi diagram.

The Delaunay triangulation of a set of sites E is a simplicial complex such that k+1 points in E form a Delaunay simplex if their Voronoi cells have nonempty intersection



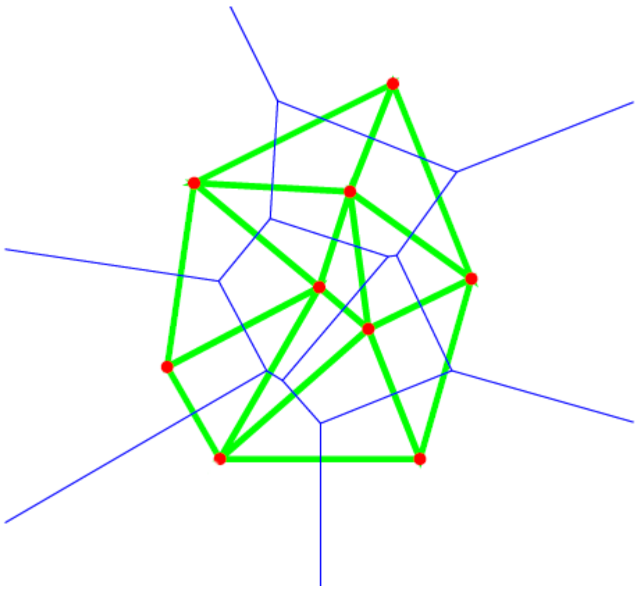
Empty Circle Property

Empty circle: A triangulation T of a point set E such that any d-simplex of T has a circumsphere that does not enclose any point of E is a Delaunay triangulation of E. Conversely, any k-simplex with vertices in E that can be circumscribed by a hypersphere that does not enclose any point of E is a face of the Delaunay triangulation of E.

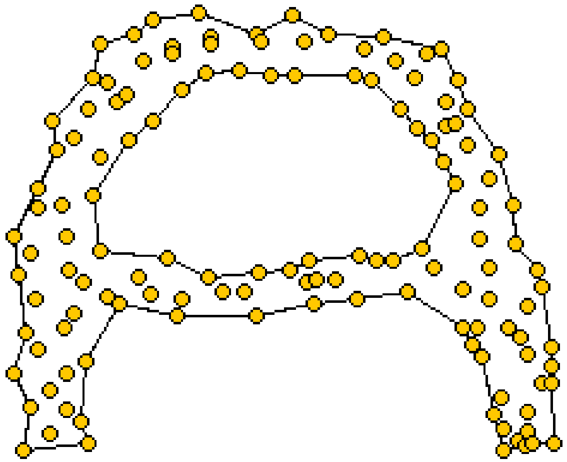


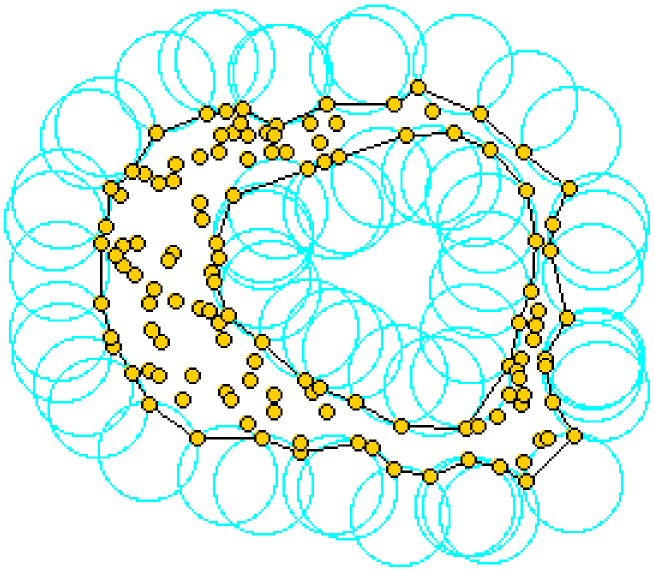
Delaunay-based

Key idea: assuming dense enough sampling, reconstructed triangles are Delaunay triangles.



Alpha-Shapes [Edelsbrunner, Kirkpatrick, Seidel]





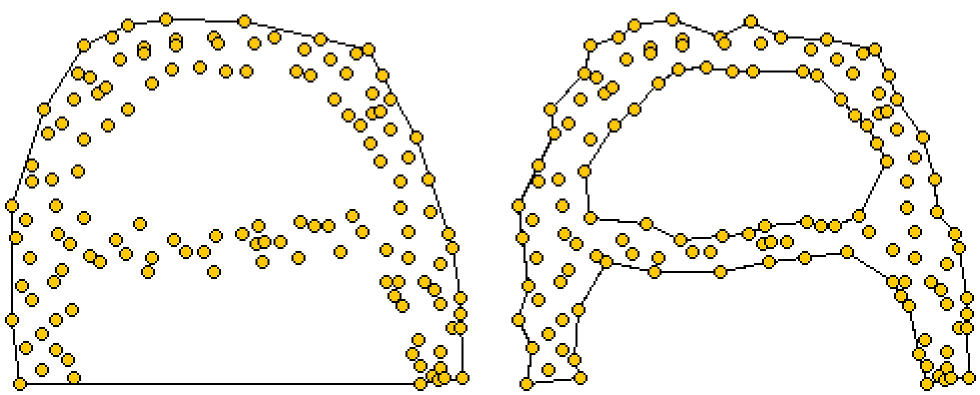
Segments: point pairs that can be touched
by an empty disc of radius alpha.

Alpha-Shapes

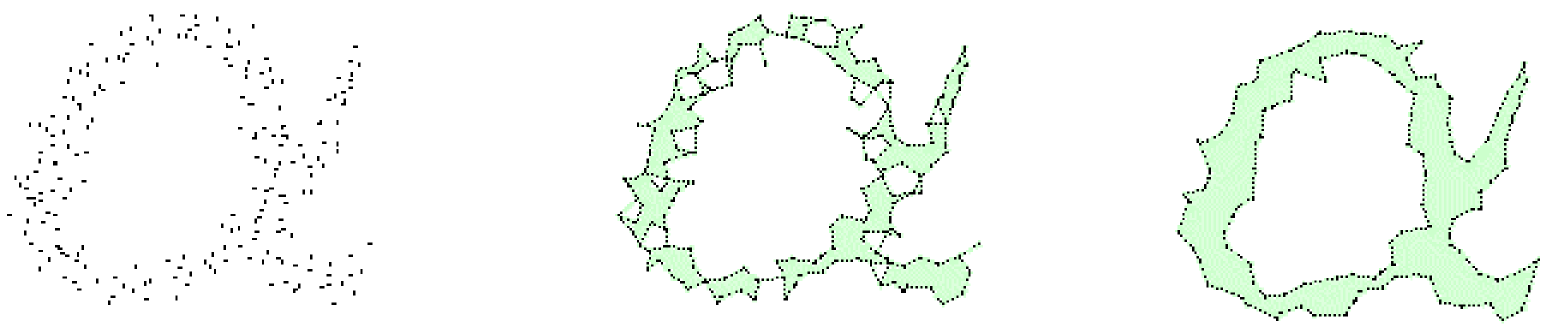
In 2D: family of piecewise linear simple curves constructed from a point set P.

Subcomplex of the Delaunay triangulation of P.

Generalization of the concept of the convex hull.

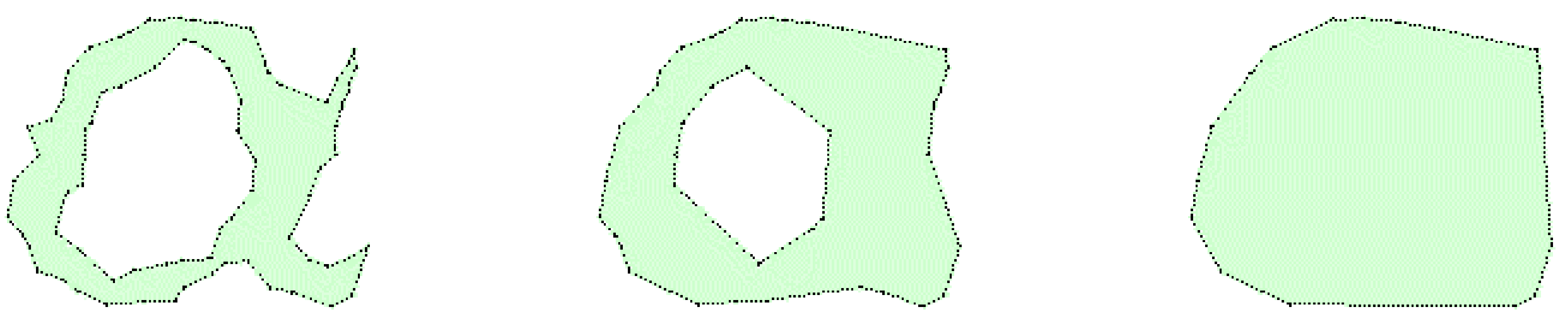


Alpha-Shapes



$\alpha = 0$

Alpha controls the desired level of detail.



$\alpha = \infty$

Delaunay-based

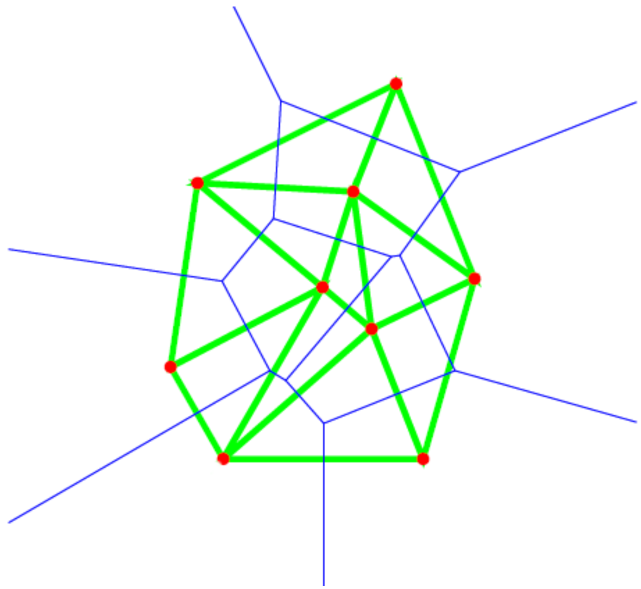
Key idea: assuming dense enough sampling, reconstructed triangles are Delaunay triangles.

First define

Medial axis

Local feature size

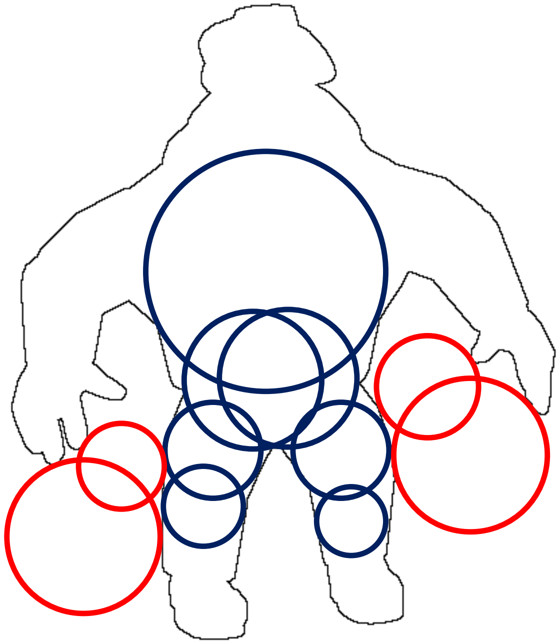
Epsilon-sampling



MEDIAL AXIS

Medial Axis

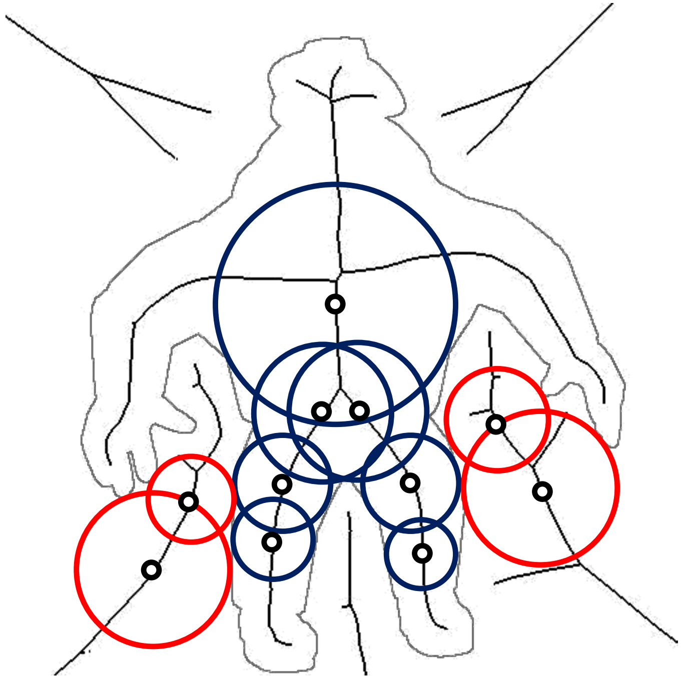
For a shape (curve/surface) a *Medial Ball* is a circle/sphere that only meets the shape tangentially, in at least two points.



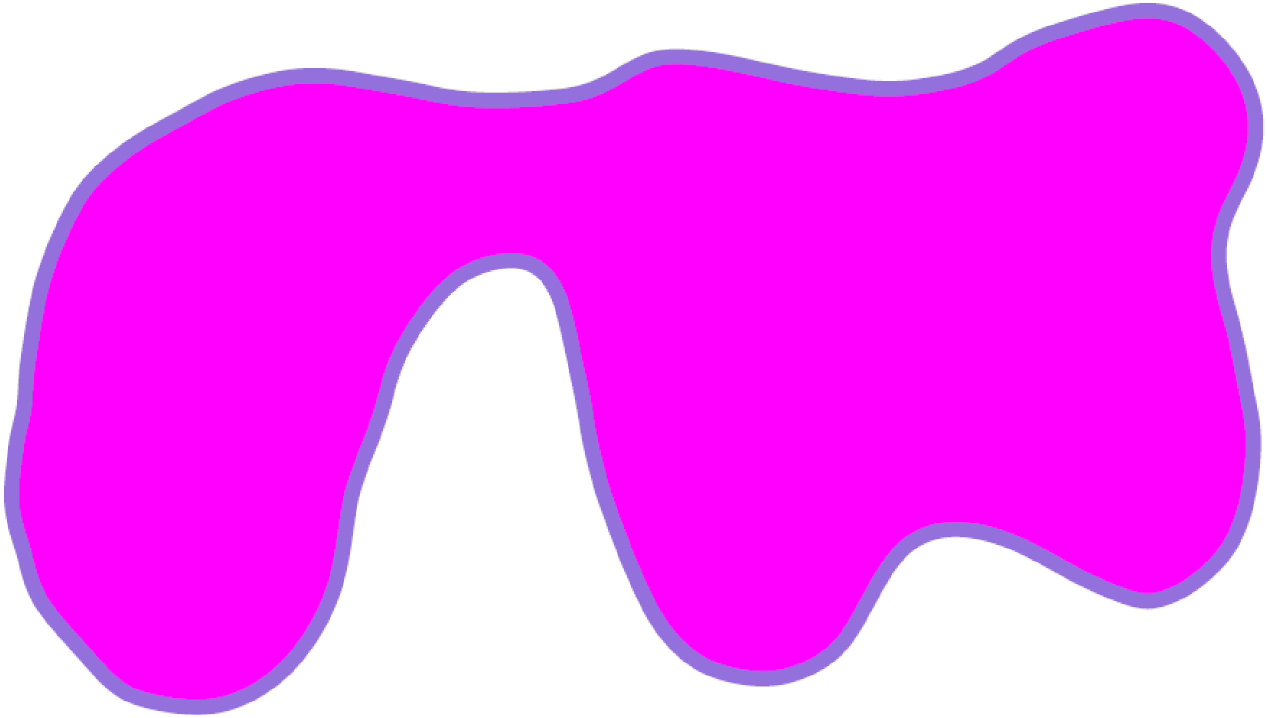
Medial Axis

For a shape (curve/surface) a *Medial Ball* is a circle/sphere that only meets the shape tangentially, in at least two points.

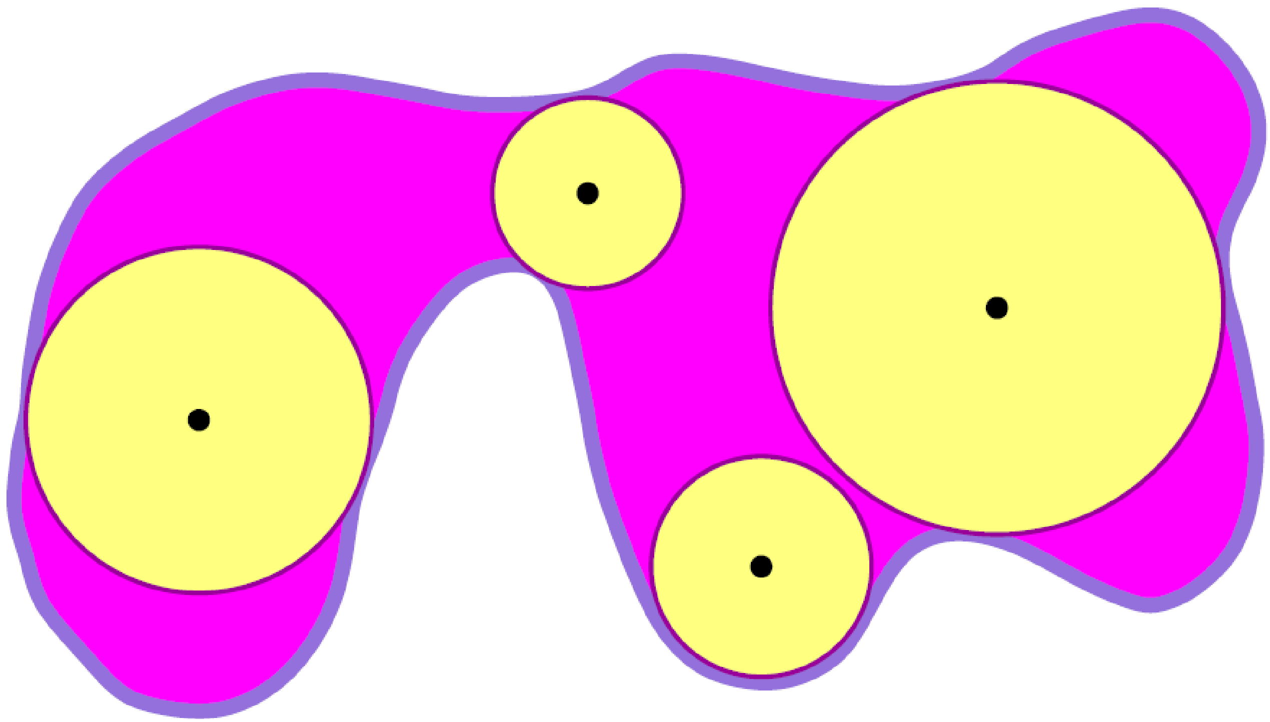
The centers of all such balls make up the *medial axis/skeleton*.



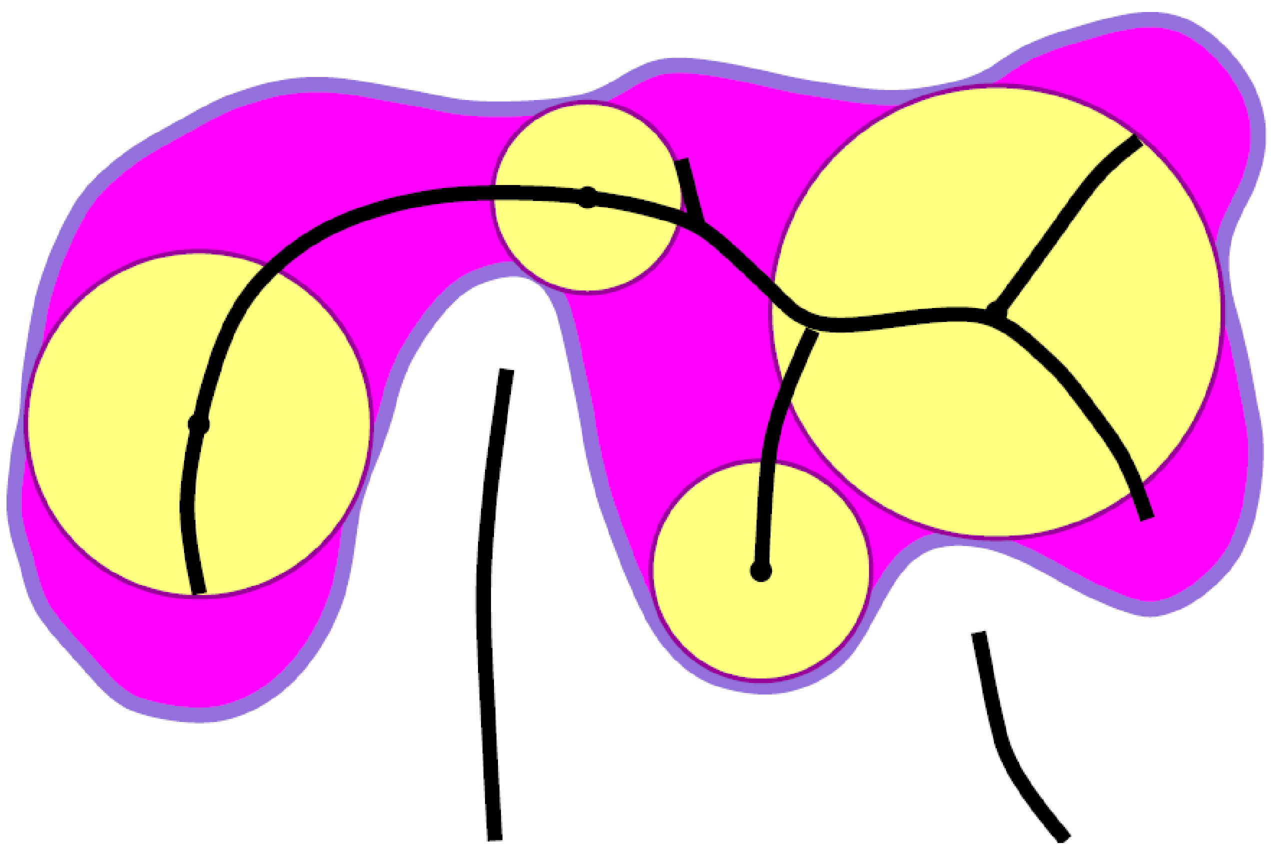
Medial Axis



Medial Axis



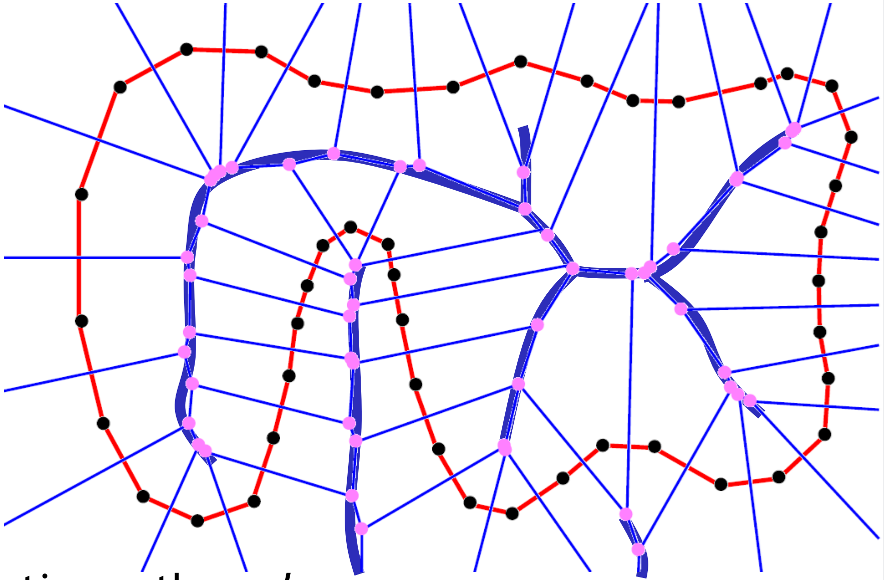
Medial Axis



Medial Axis

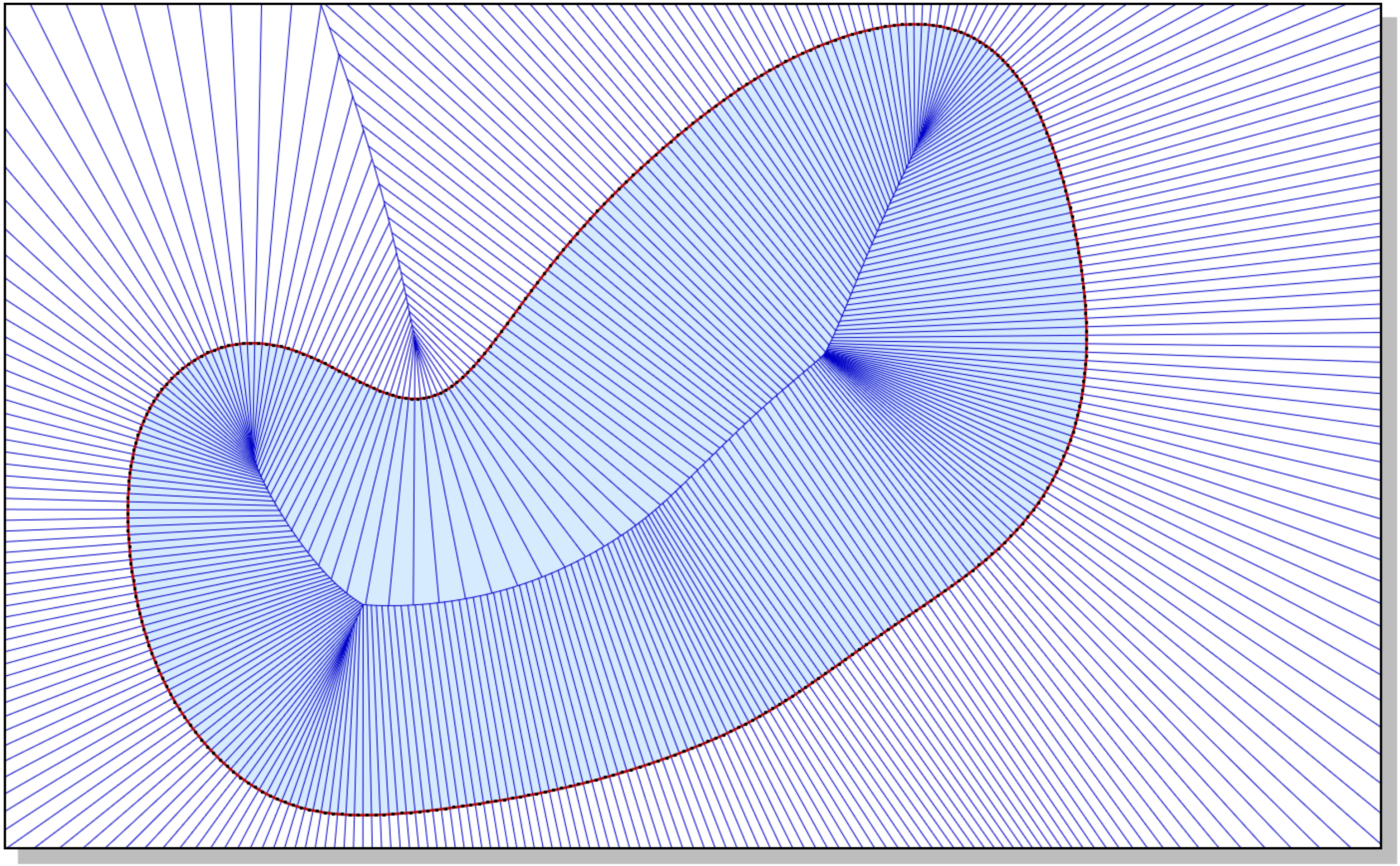
Observation*:

For a reasonable point sample, the medial axis is well-sampled by the Voronoi vertices.

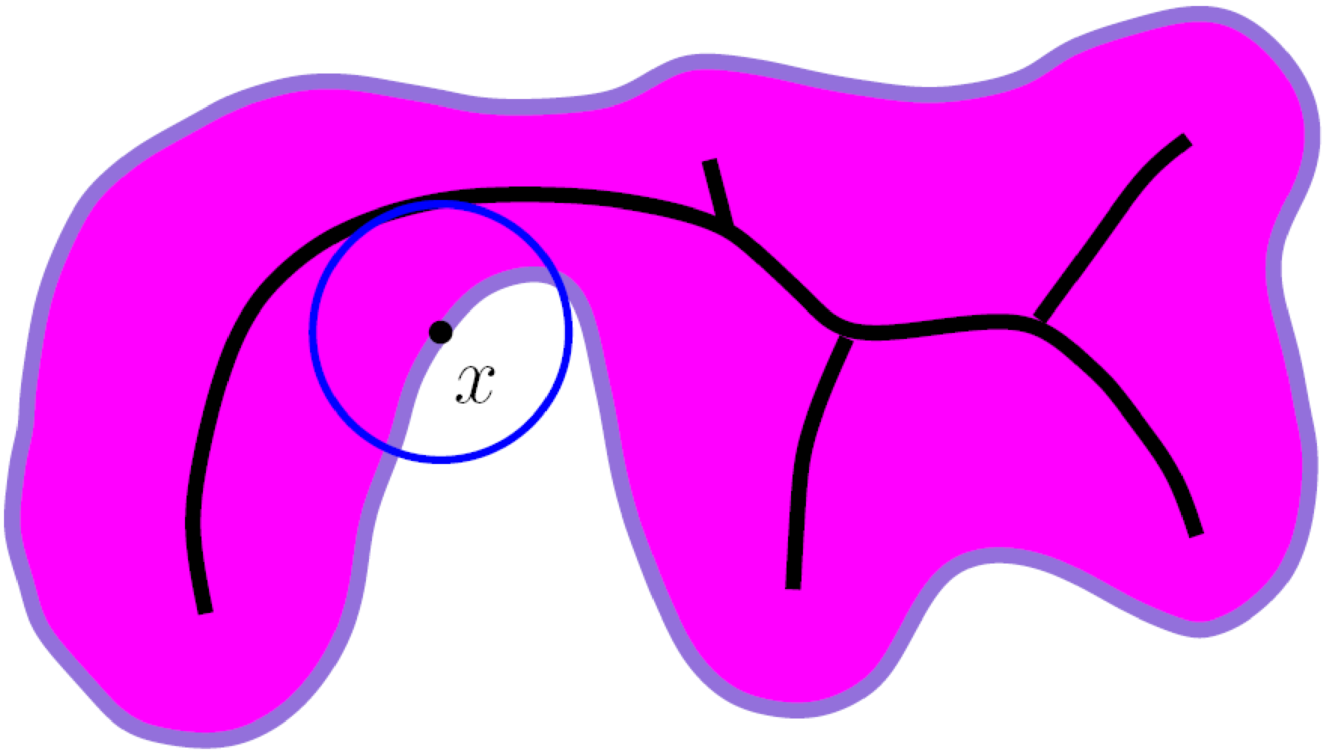


*In 3D, this is only true for a subset of the Voronoi vertices – the *poles*.

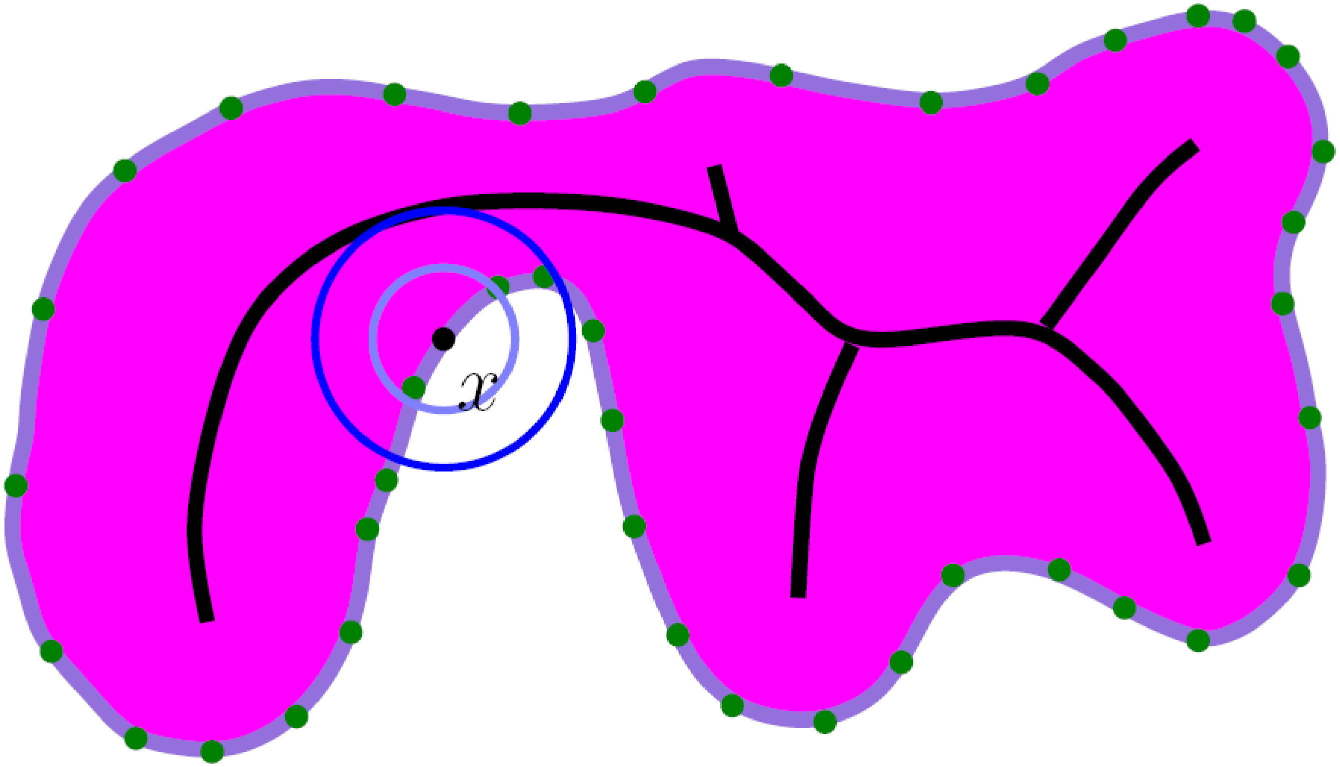
Voronoi & Medial Axis



Local Feature Size



Epsilon-Sampling



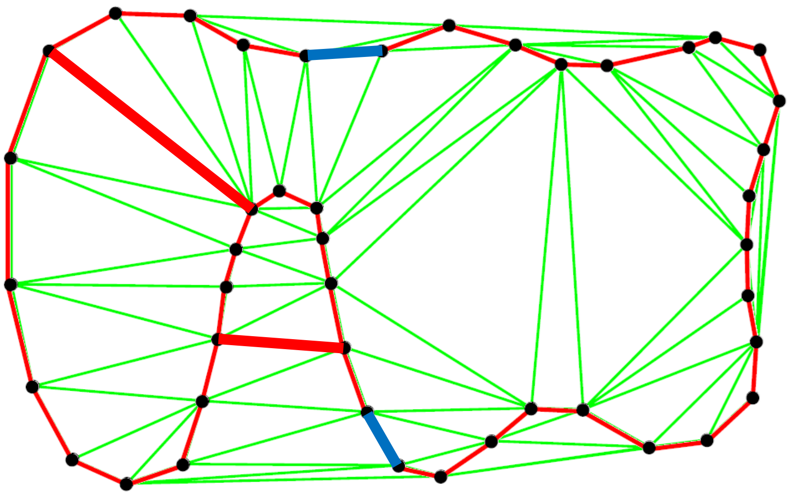
Crust [Amenta et al. 98]

If we consider the Delaunay Triangulation of a point set, the shape boundary can be described as a subset of the Delaunay edges.

Q: How do we determine which edges to keep?

A: Two types of edges:

1. Those connecting adjacent points on the boundary
2. Those traversing the shape.



Discard those that traverse.

Crust [Amenta et al. 98]

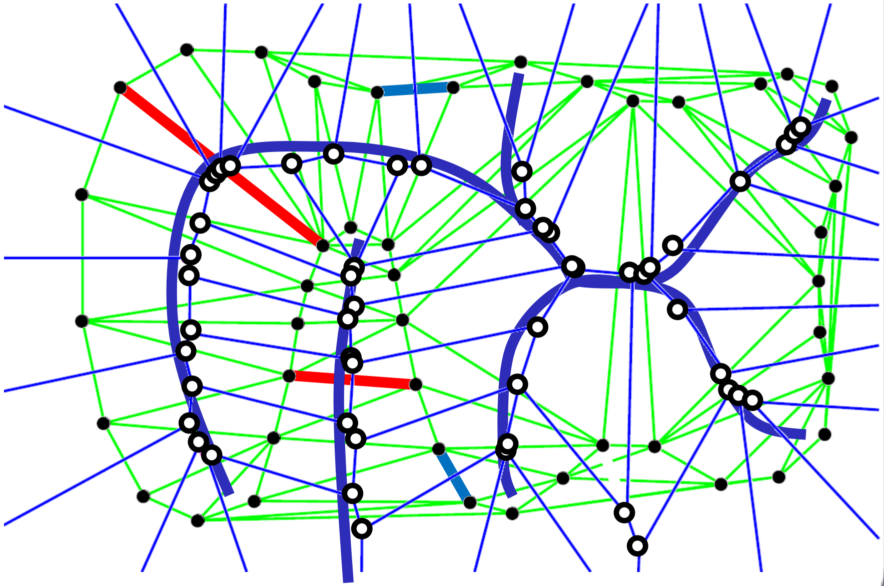
Observation:

Edges that traverse cross the medial axis.

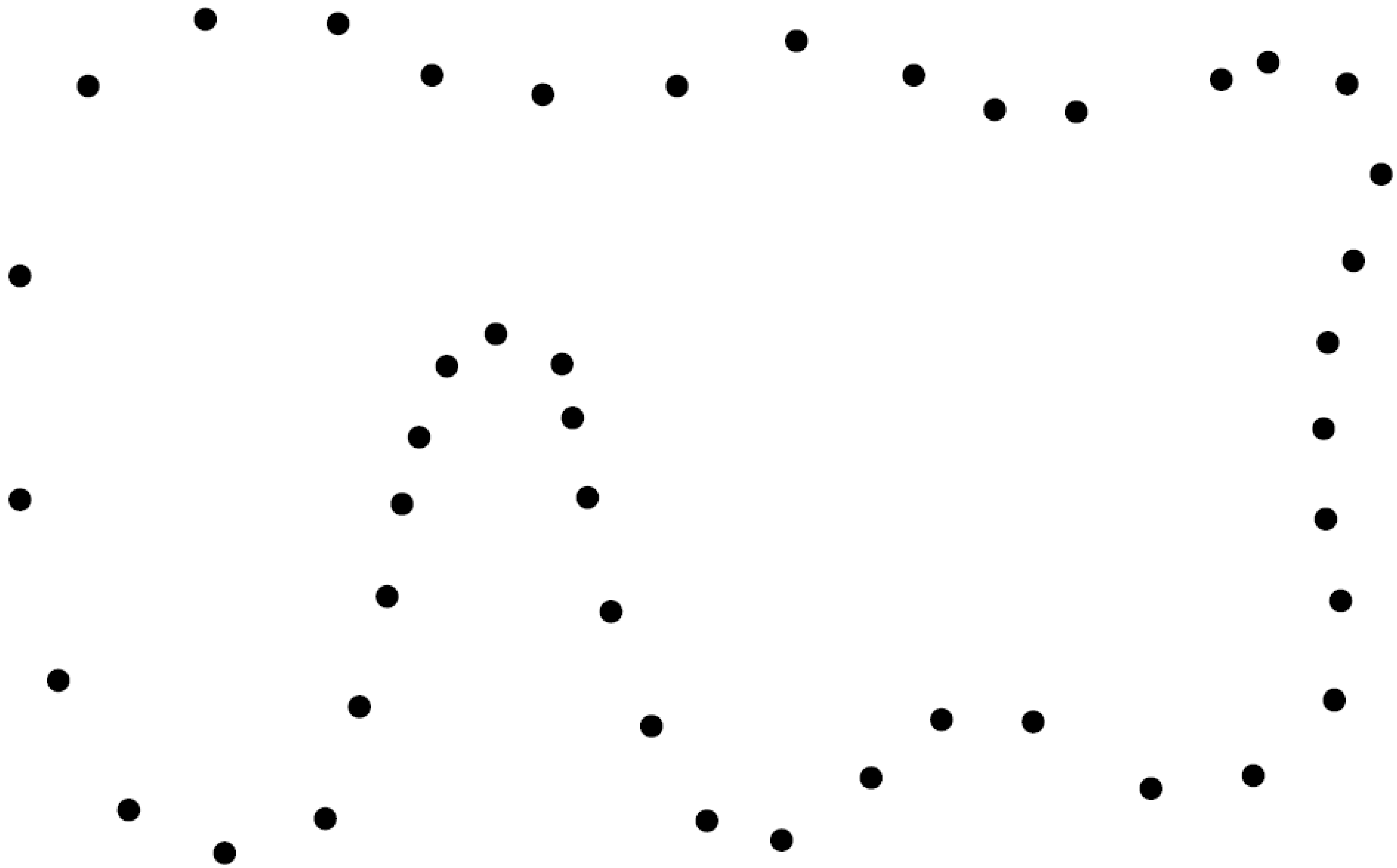
Although we don't know the axis, we can sample it with the Voronoi vertices.

Edges that traverse must

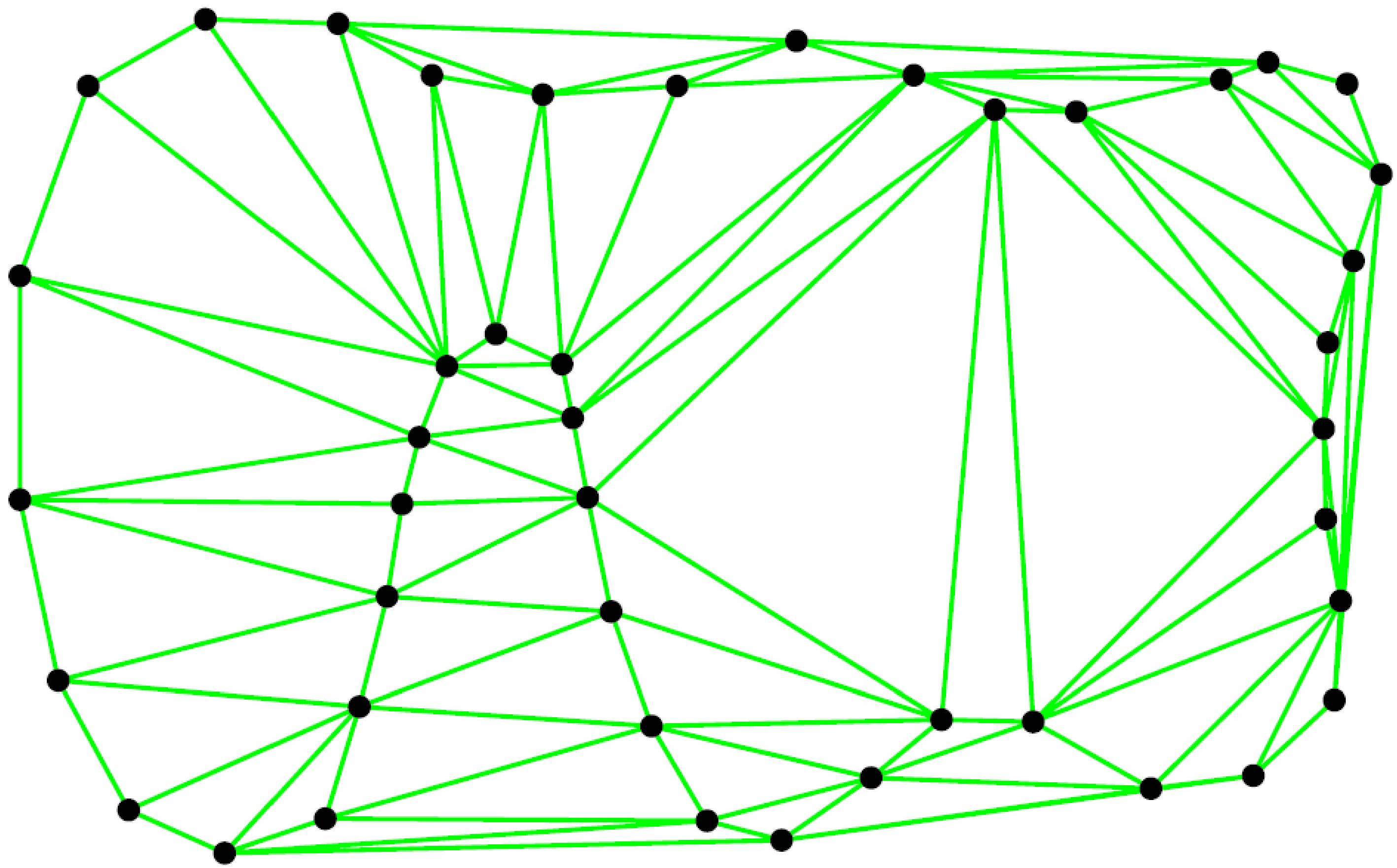
be near the Voronoi vertices.



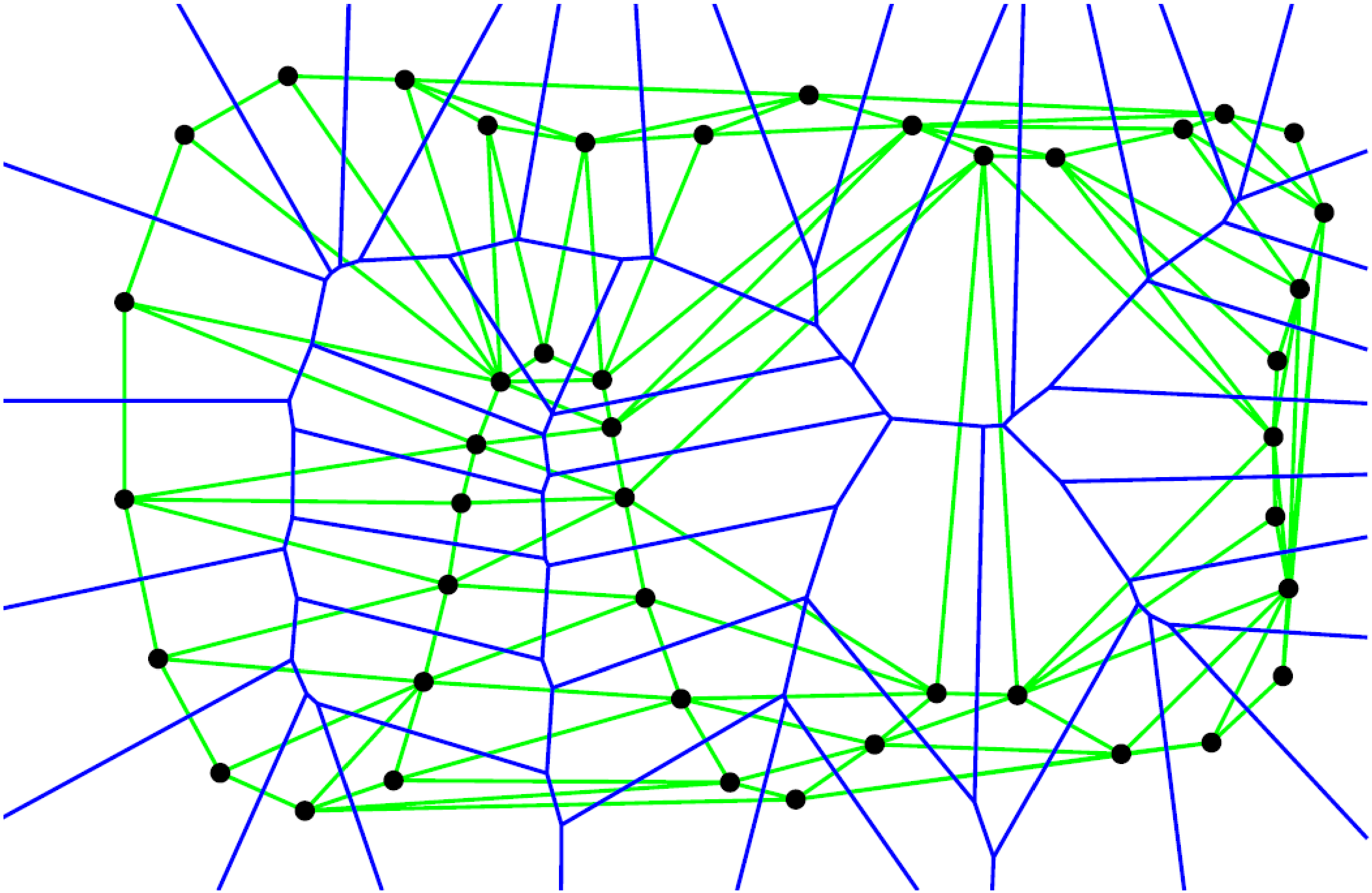
Crust [Amenta et al. 98]



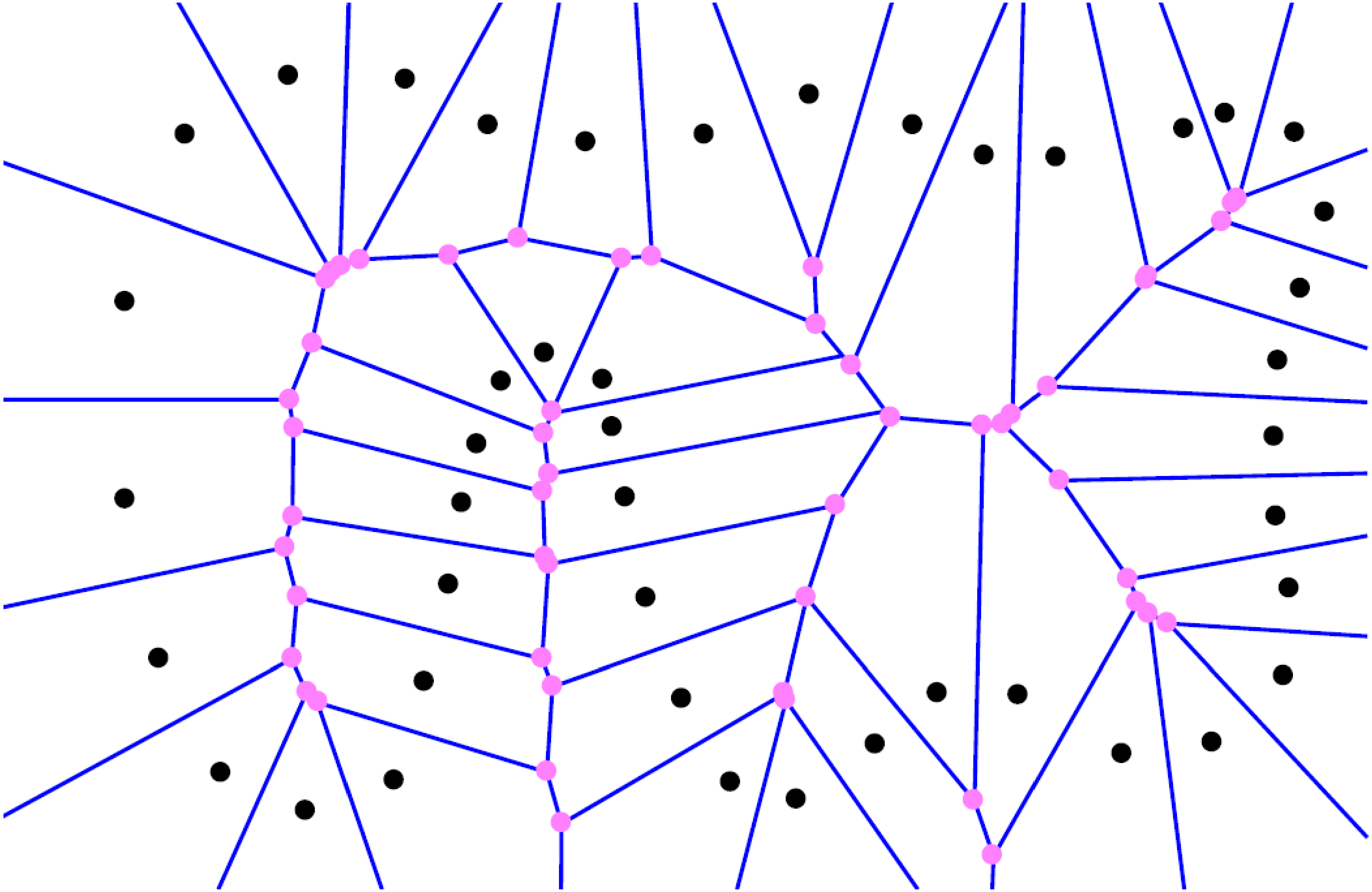
Delaunay Triangulation



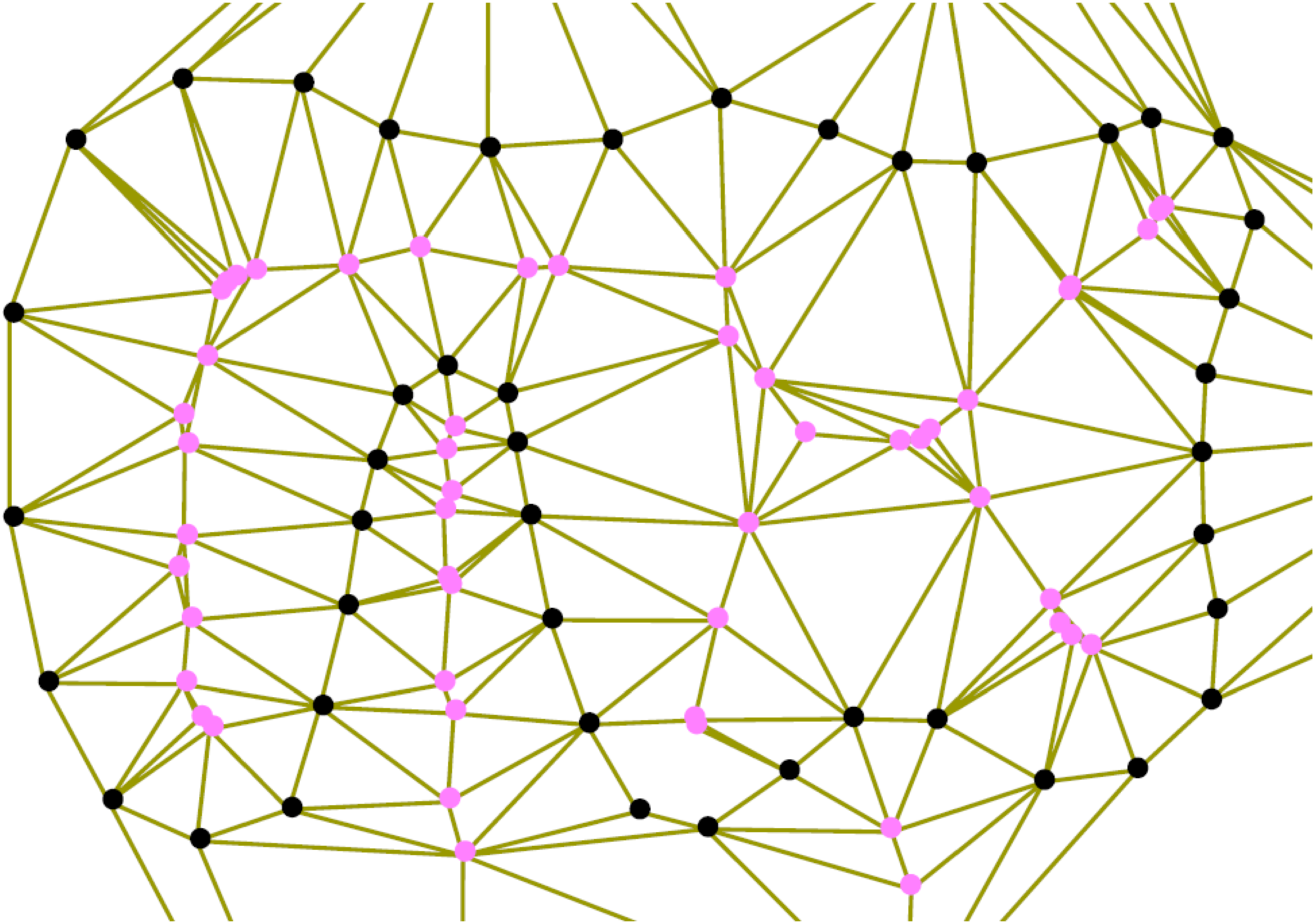
Delaunay Triangulation & Voronoi Diagram



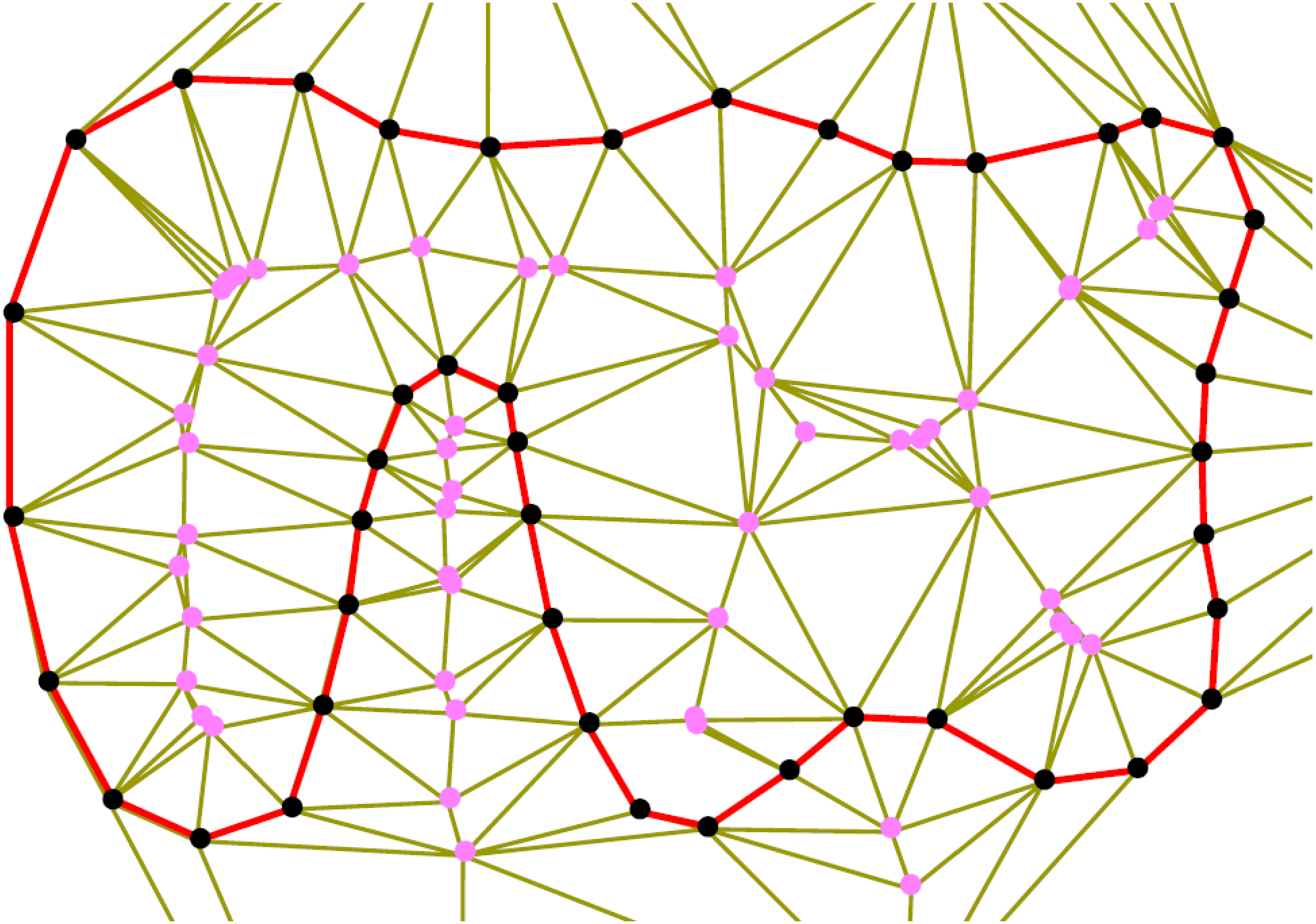
Voronoi Vertices



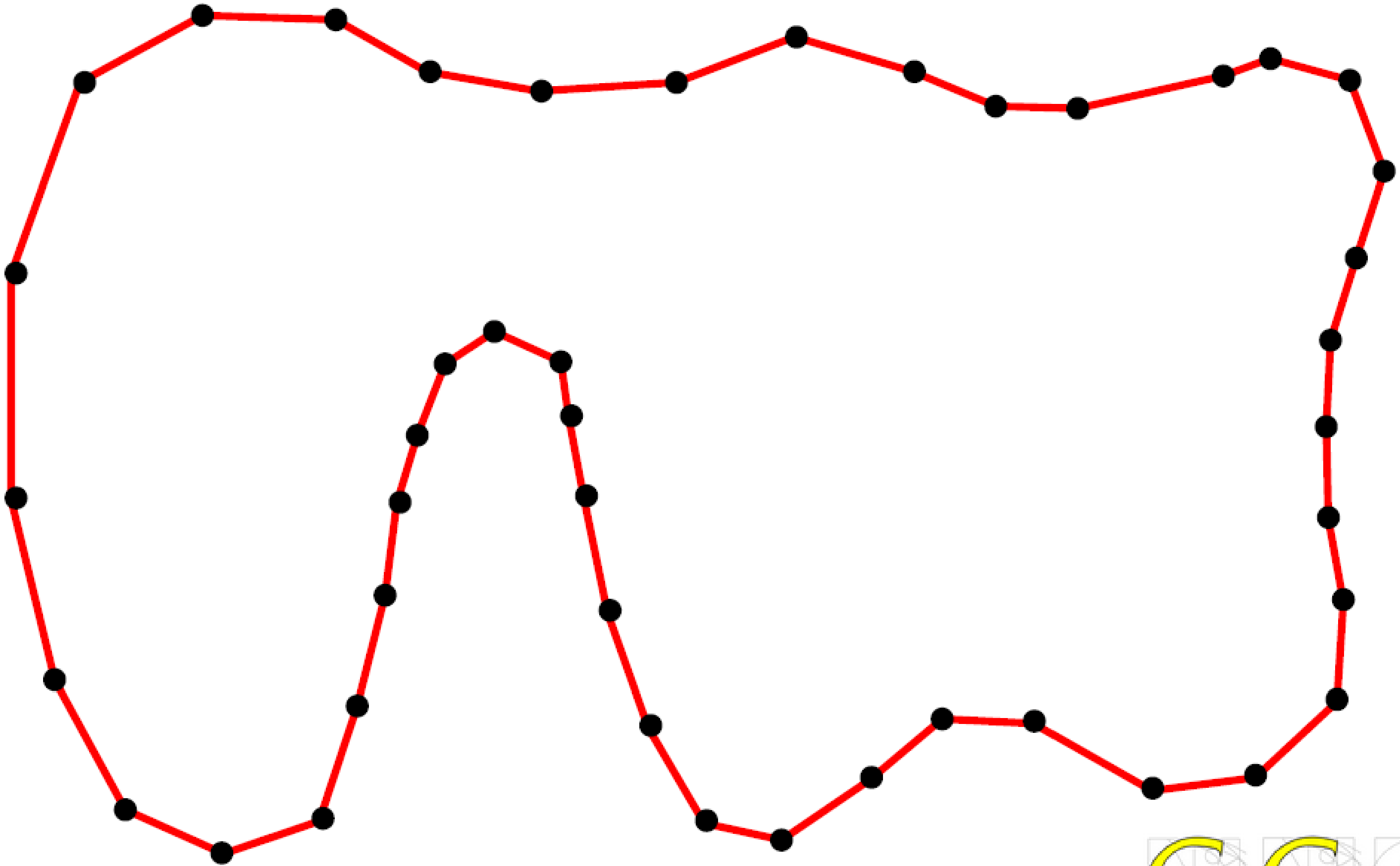
Refined Delaunay Triangulation



Crust



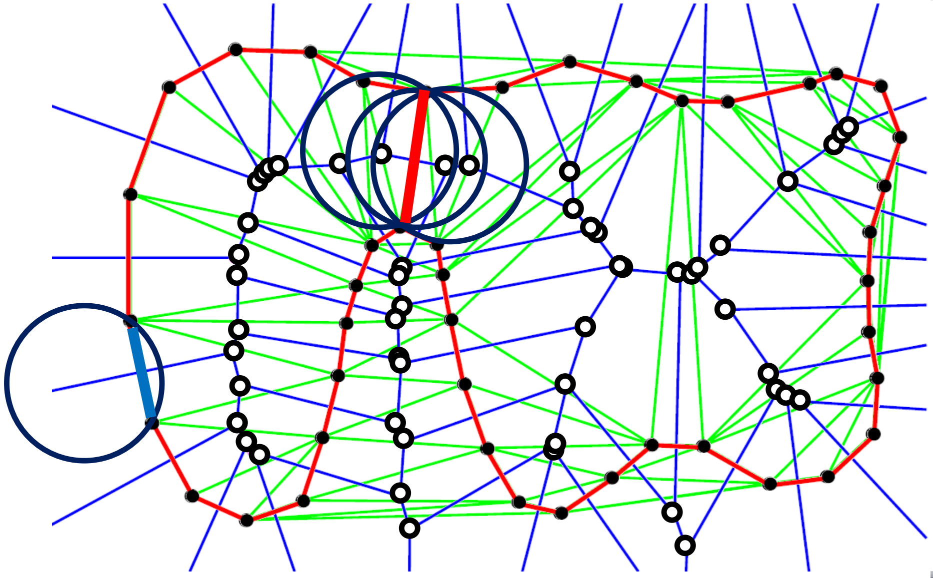
Crust



Crust (variant)

Algorithm:

1. Compute the Delaunay triangulation.
2. Compute the Voronoi vertices
3. Keep all edges for which there is a circle that contains the edge but no Voronoi vertices.

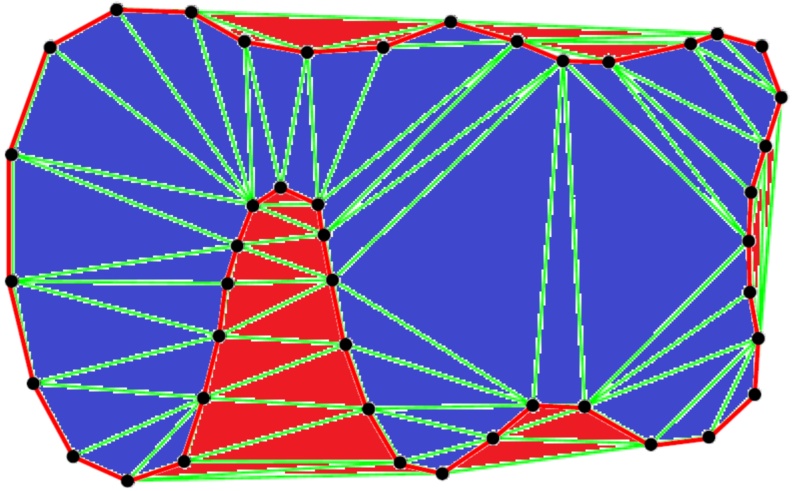


SPECTRAL « CRUST »

Space Partitioning

Given a set of points, construct the Delaunay triangulation.

If we label each triangle as inside/outside, then the surface of interest is the set of edges that lie between inside and outside triangles.

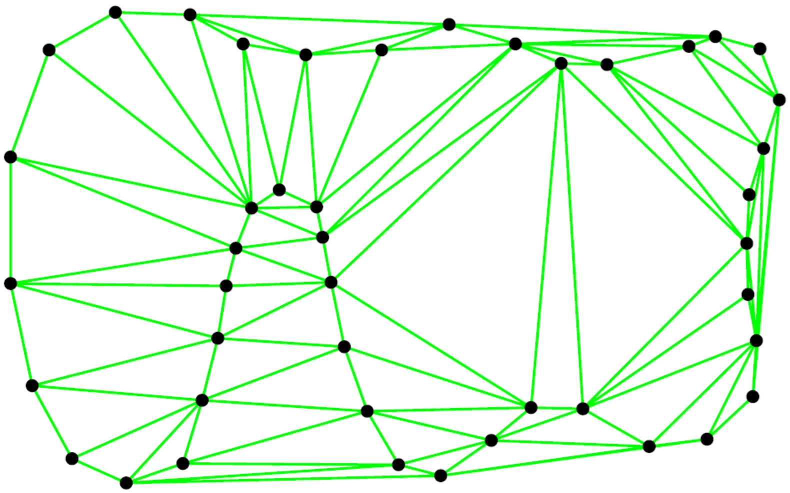


Space Partitioning

Q: How to assign labels?

A: Spectral Partitioning

Assign a weight to each edge indicating if the two triangles are likely to have the same label.



[Kolluri et al., 2004]

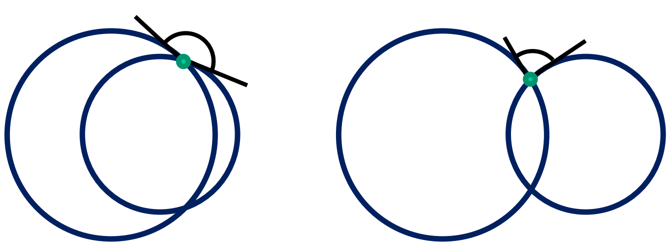
Space Partitioning

<u>Assigning edge weights</u>

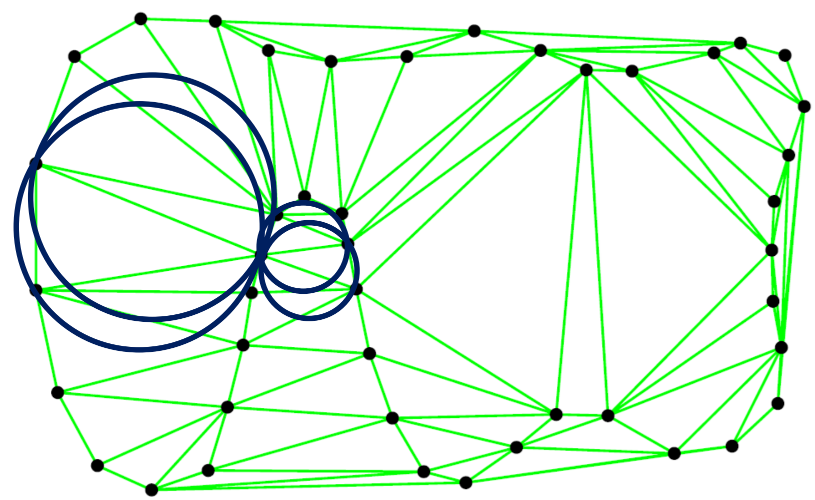
Q: When are triangles on opposite sides of an edge likely to have the same label?

A: If the triangles are on the same side, their circumscribing circles intersect <u>deeply</u>.

Use the angle of intersection to set the weight.

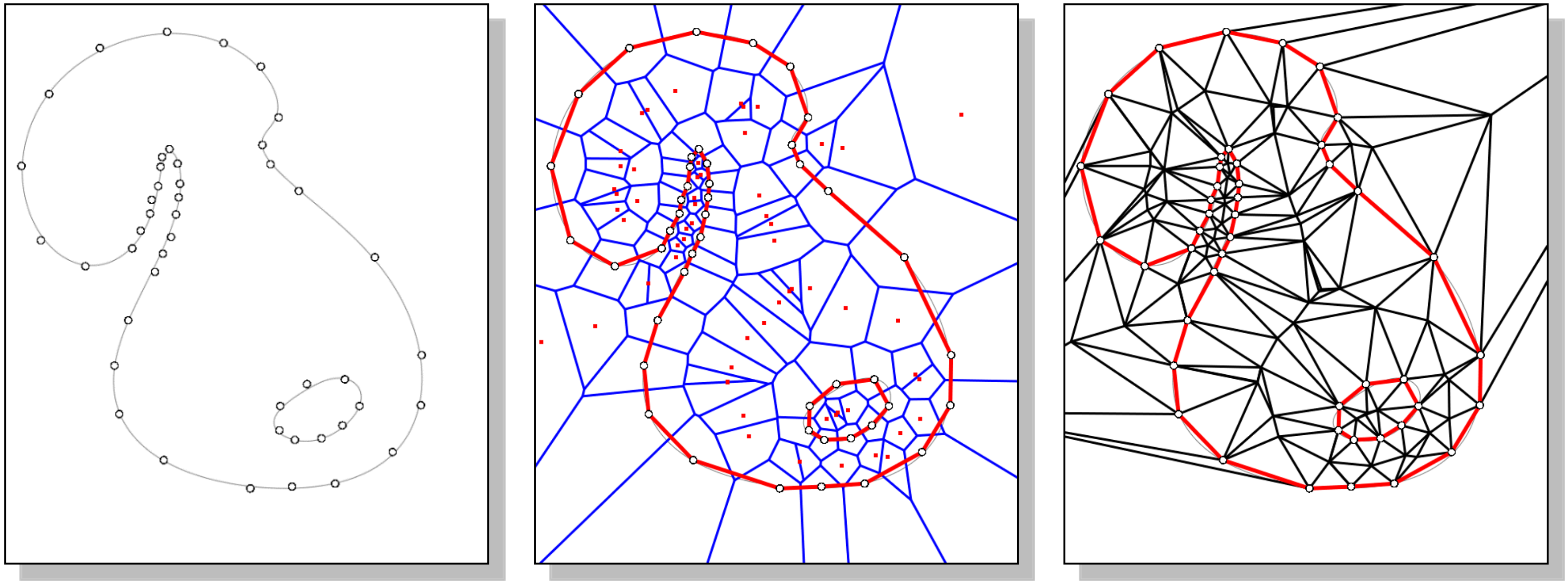


Large Weight Small Weight



Crust

Several Delaunay algorithms provably correct

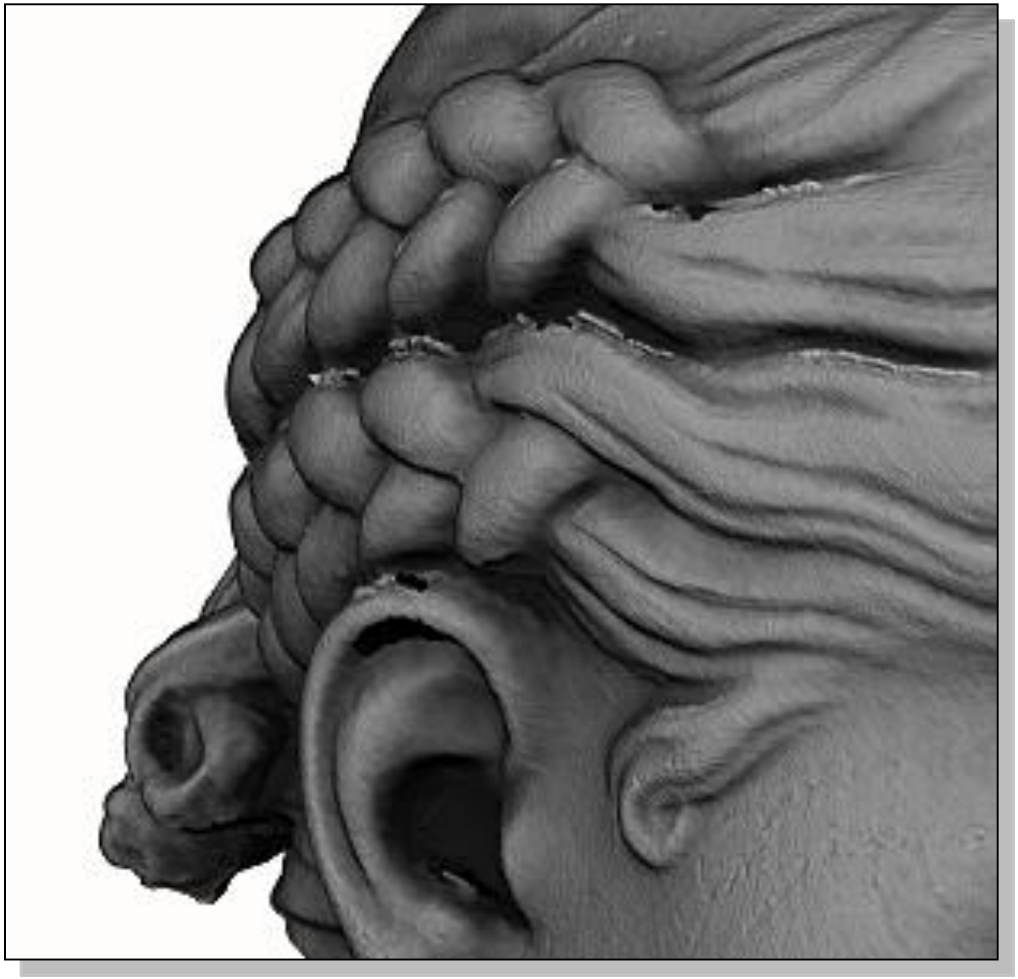


Delaunay-based

Several Delaunay algorithms are **provably correct**... in the absence of noise and undersampling.

perfect data ?

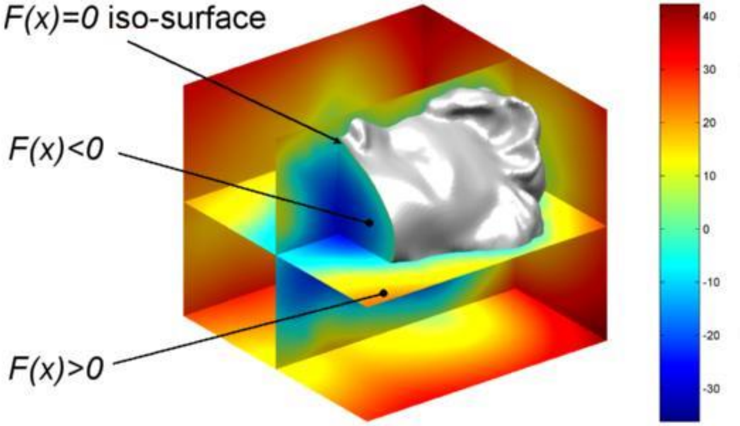
Noise & Undersampling



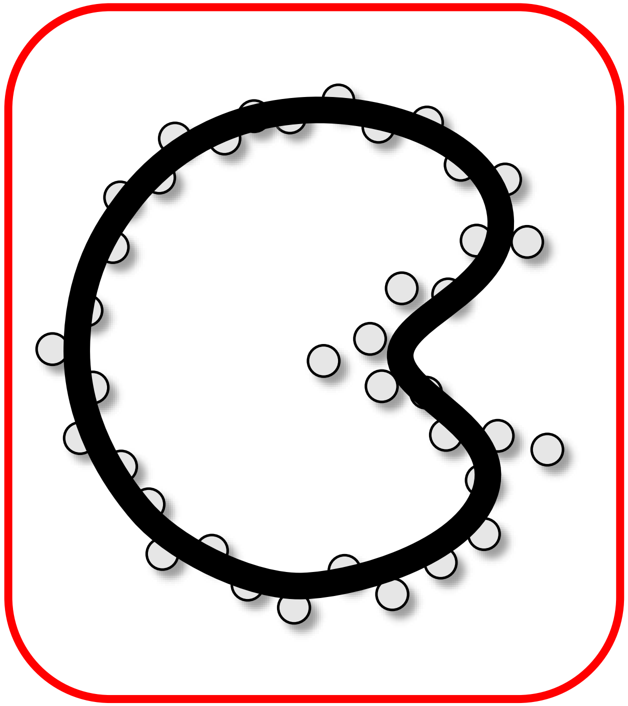
Delaunay-based

Several Delaunay algorithms are **provably correct**… in the absence of noise and undersampling.

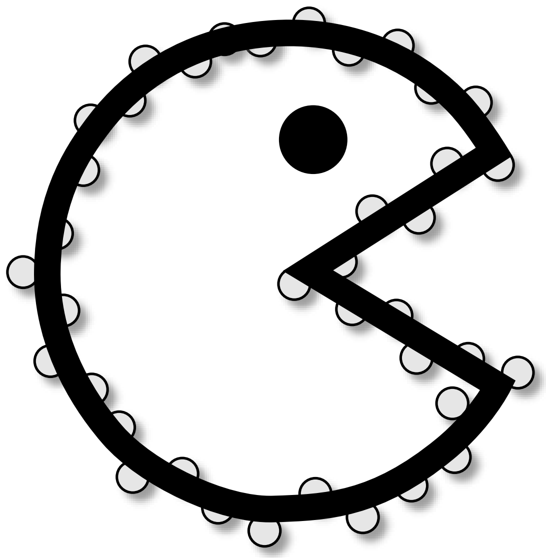
Motivates reconstruction by fitting **approximating** implicit surfaces



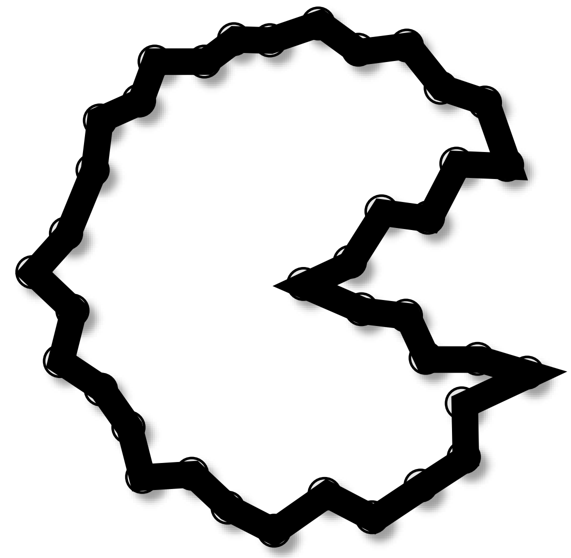
VARIATIONAL FORMULATIONS



Smooth



Piecewise Smooth



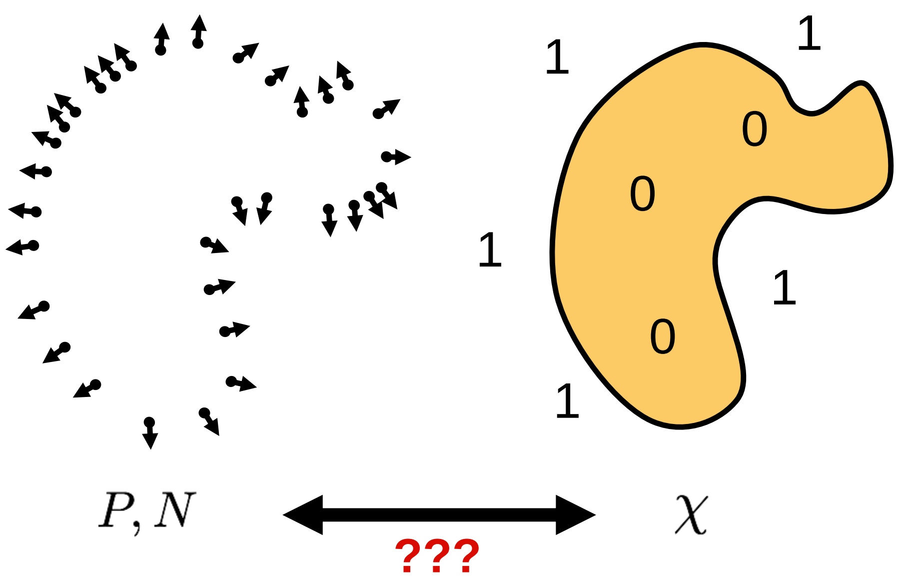
“Simple”

Poisson Surface Reconstruction

[**Kazhdan et al.** 06]

Indicator Function

Construct indicator function from point samples



Indicator Function

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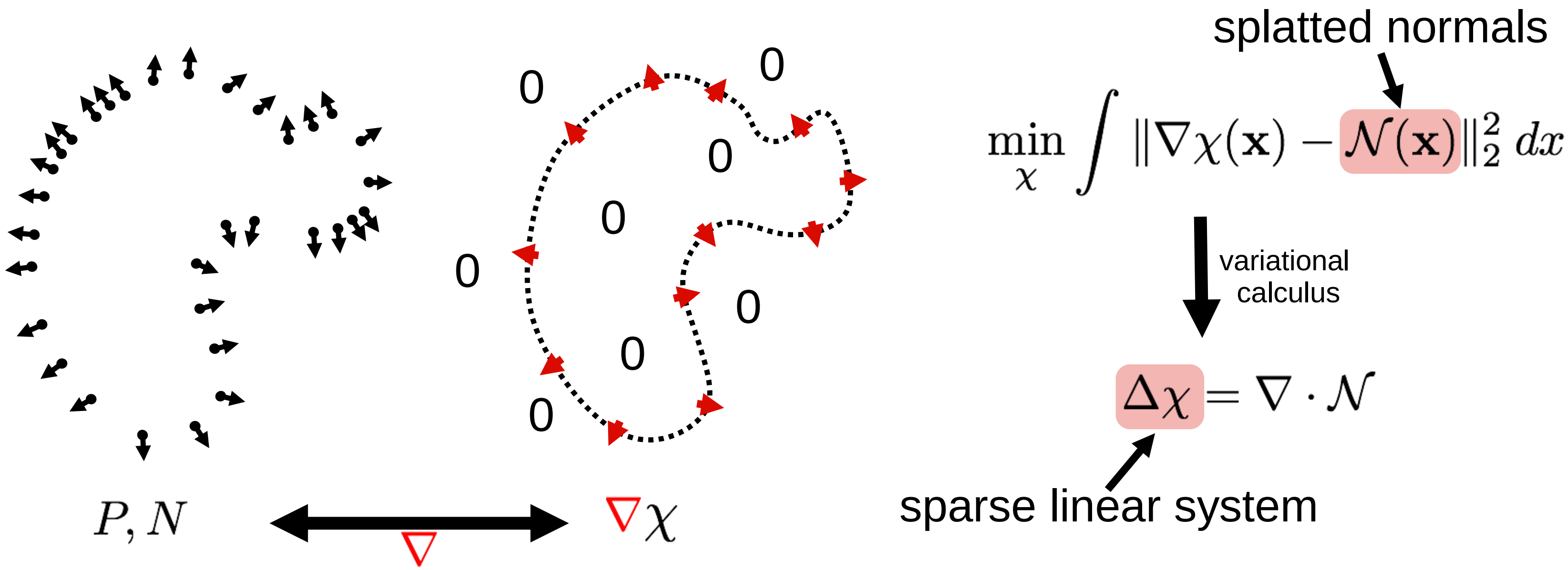
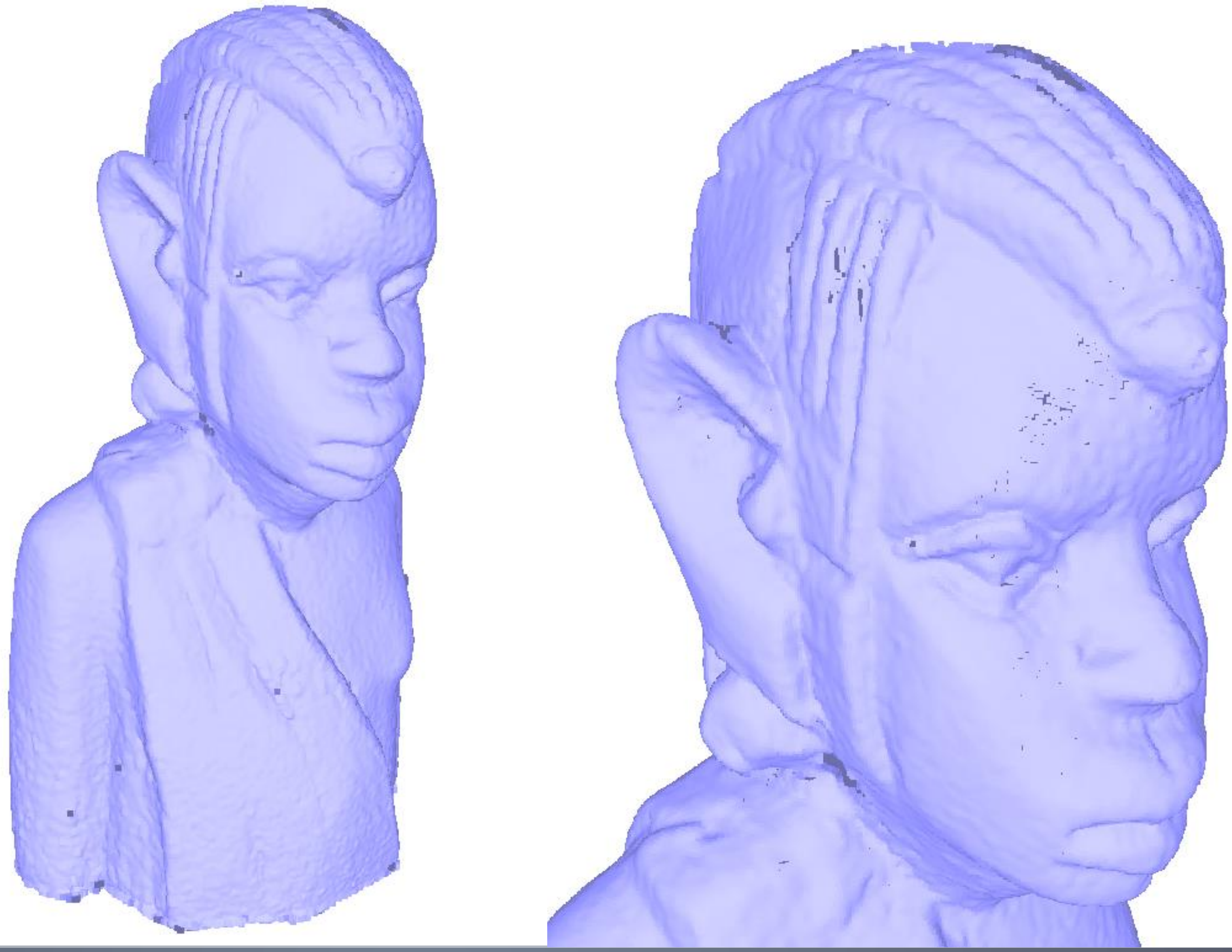
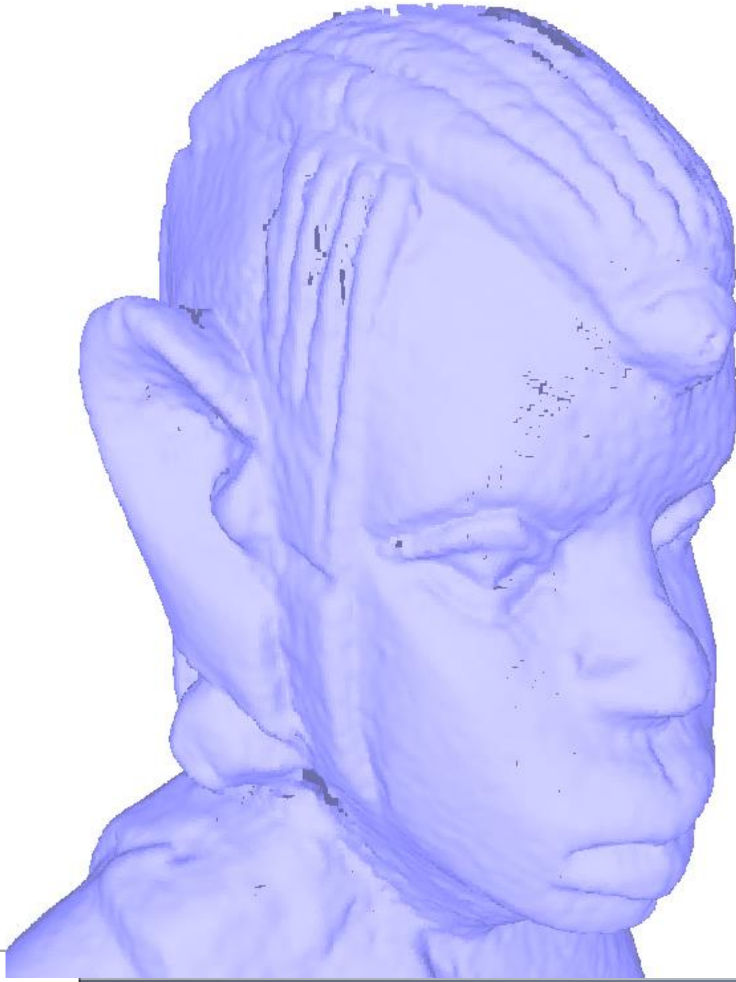
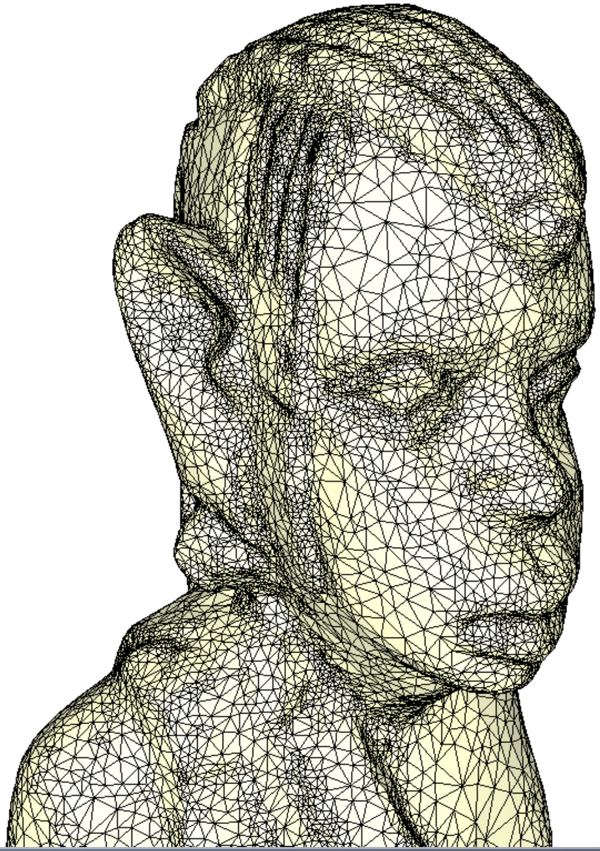


Poisson Surface Reconstruction



Poisson Surface Reconstruction





WHAT NEXT

What Next

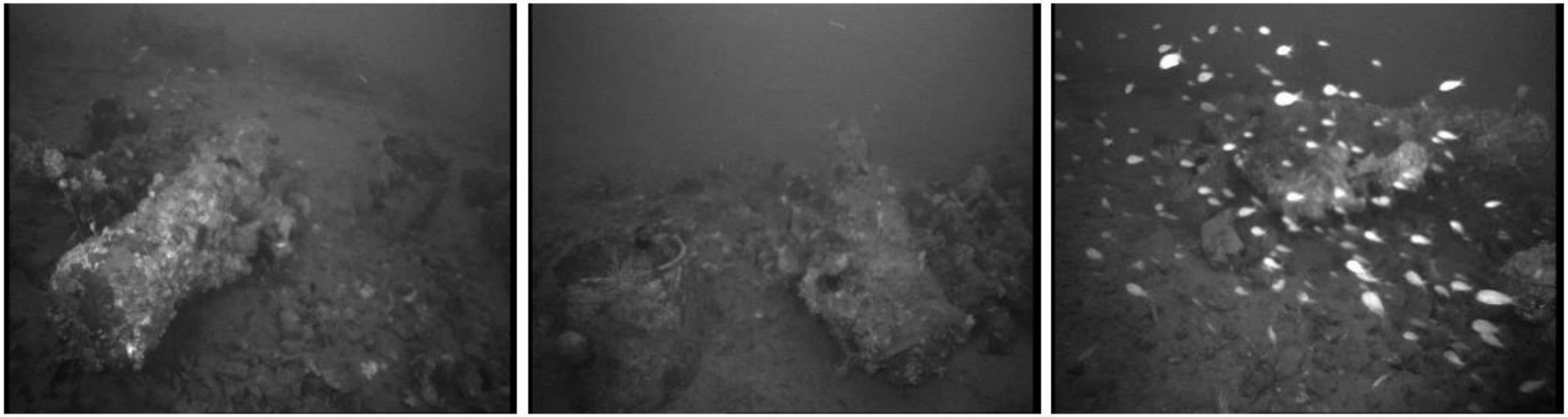
Online

- Reconstruction
- Localization

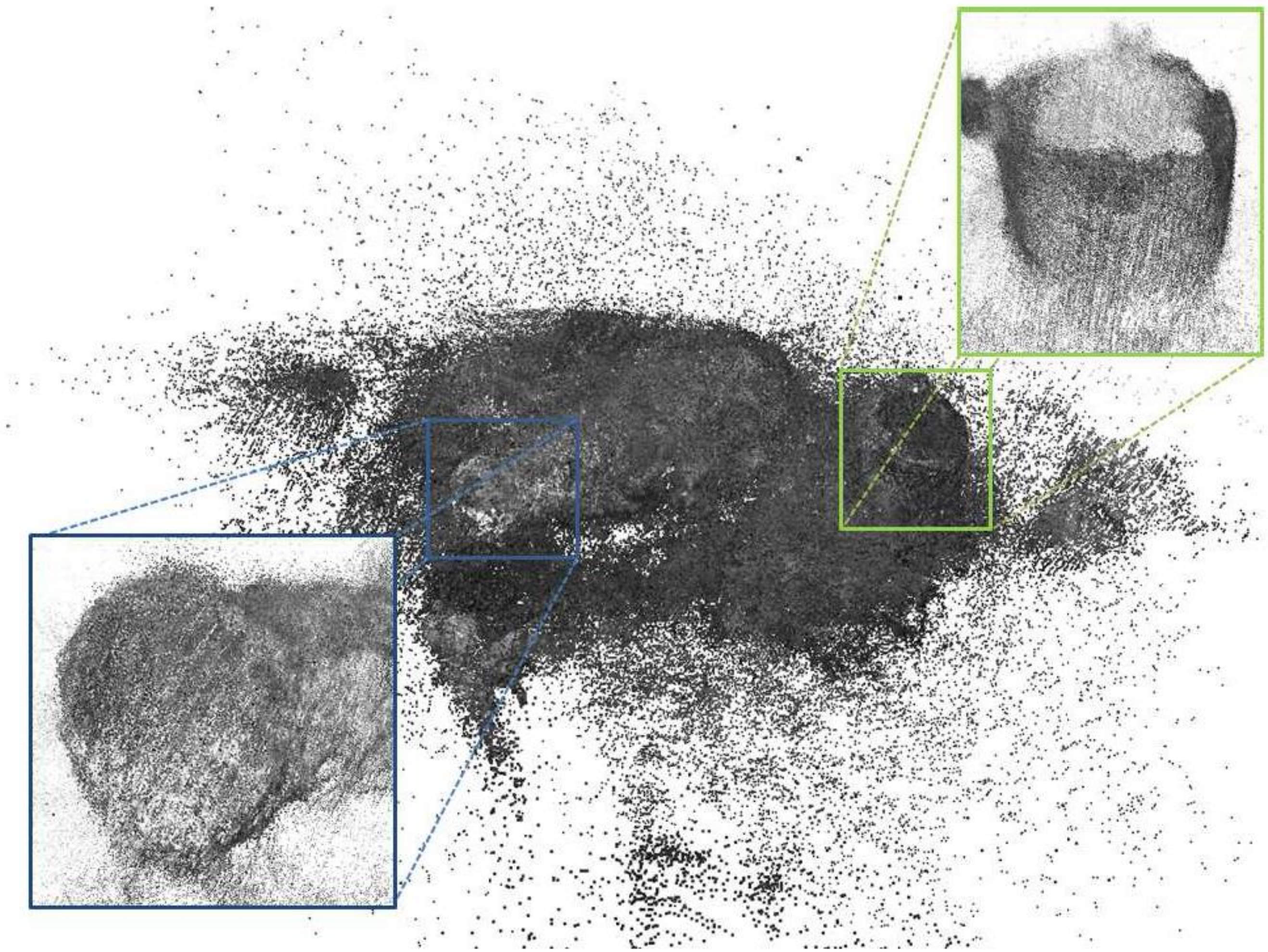
Robustness

- Structured outliers
- Heterogeneous data

« La Lune »



« La Lune »



Simplification & Approximation

Pierre Alliez
Inria Sophia Antipolis

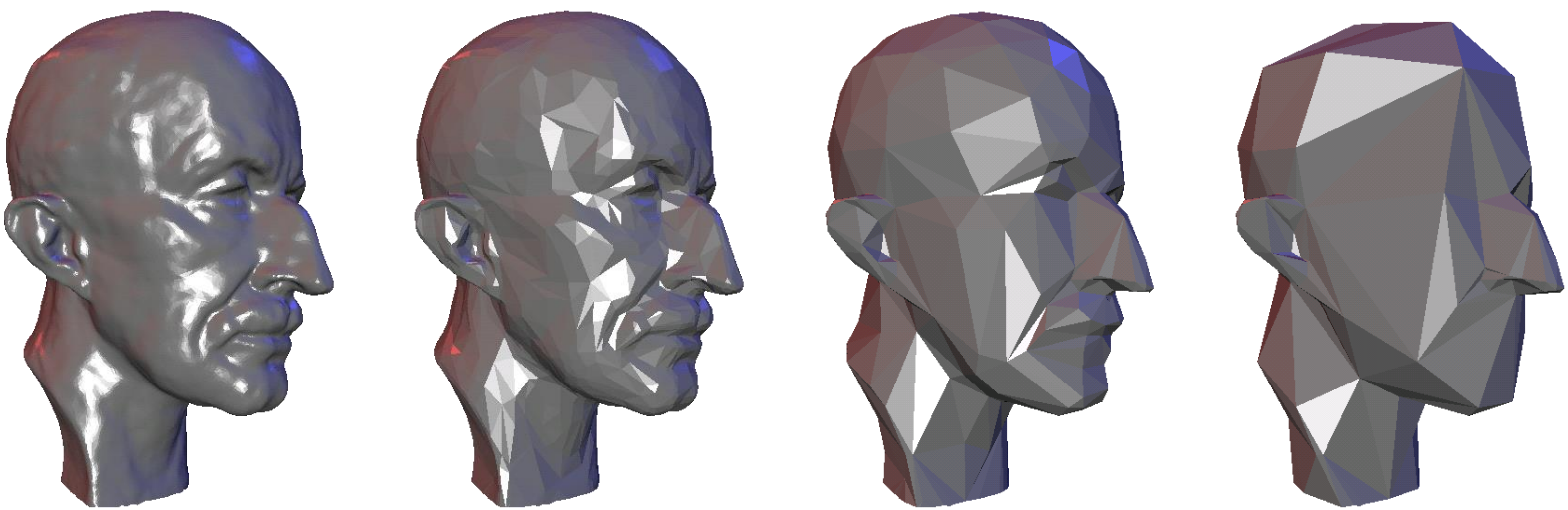
Inria
informatics mathematics

Outline

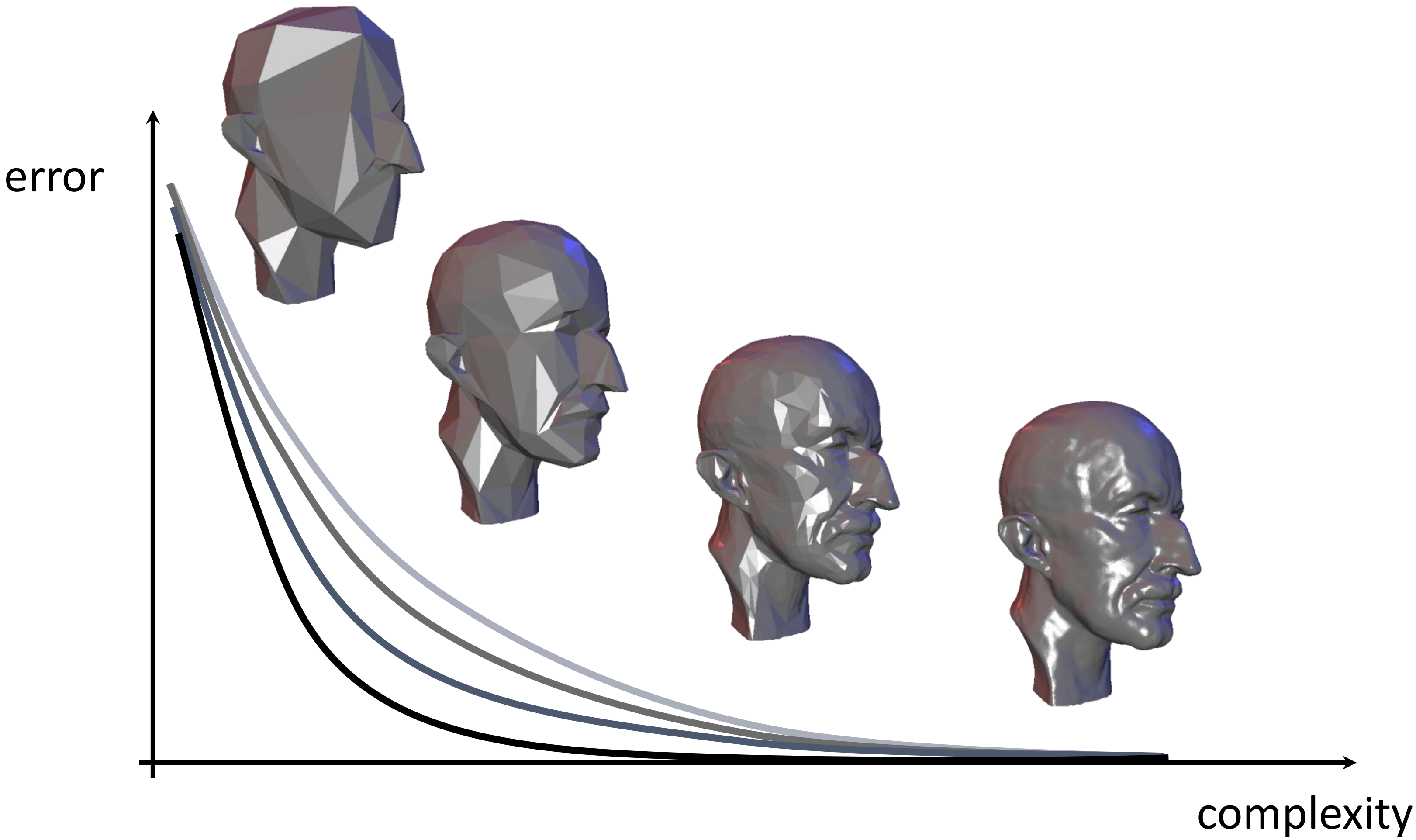
- Motivations
- Simplification
- Approximation
- Remaining Challenges

Motivations

- Multi-resolution hierarchies for
  - efficient geometry processing
  - level-of-detail (LOD) rendering



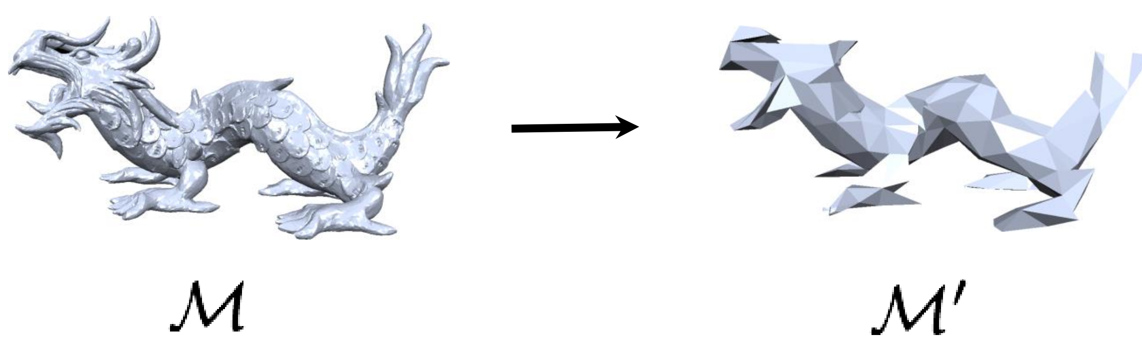
Complexity-Error Tradeoff



Problem Statement

- Given: $\mathcal{M} = (\mathcal{V}, \mathcal{F})$
- Find: $\mathcal{M}' = (\mathcal{V}', \mathcal{F}')$ such that

1. $|\mathcal{V}'| = n < |\mathcal{V}|$ and $\|\mathcal{M} - \mathcal{M}'\|$ is minimal, or
2. $\|\mathcal{M} - \mathcal{M}'\| < \epsilon$ and $|\mathcal{V}'|$ is minimal



Problem Statement

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2. $\|\mathcal{M} - \mathcal{M}'\| < \epsilon$ and $|\mathcal{V}'|$ is minimal

hard! [Agarwal-Suri 1998]

→ look for sub-optimal solution

Simplification

Simplification

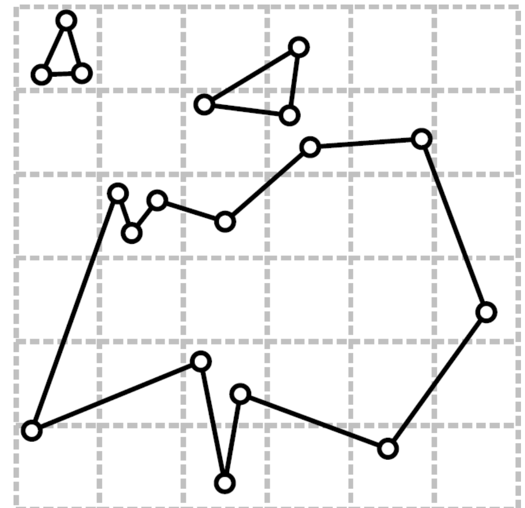
- Vertex Clustering
- Iterative Decimation
- Extensions

Simplification

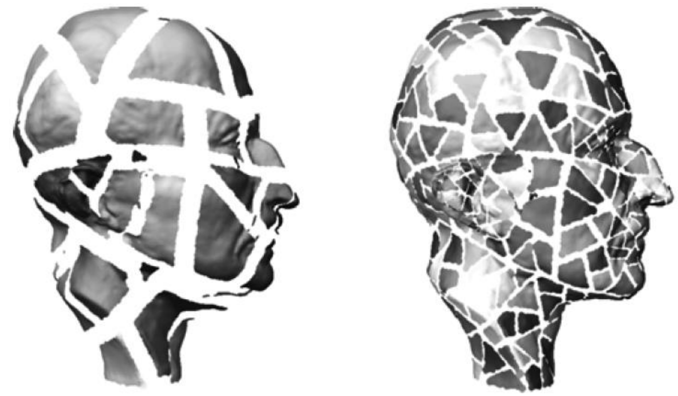
- **Vertex Clustering**
- Iterative Decimation
- Extensions

Vertex Clustering

- Cluster Generation
  - Uniform 3D grid
  - Map vertices to cluster cells
- Computing a representative
- Mesh generation
- Topology changes



Vertex Clustering

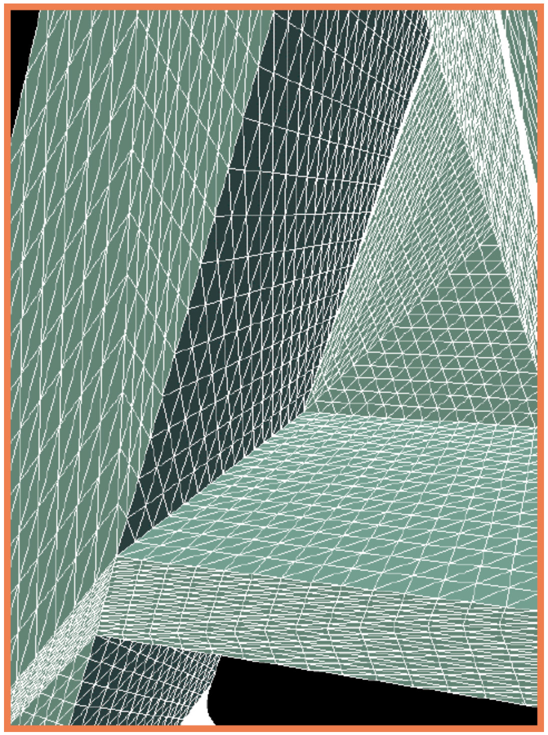


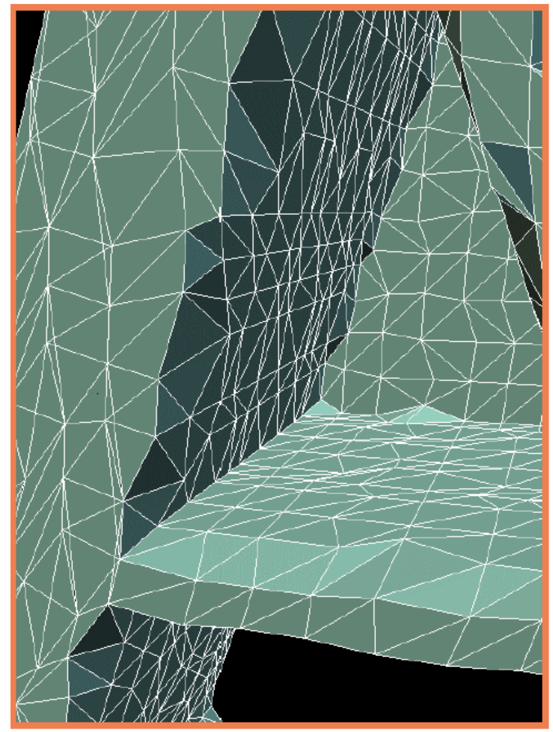
- Cluster Generation
  - Hierarchical approach
  - Top-down or bottom-up
- Computing a representative
- Mesh generation
- Topology changes

Vertex Clustering

- Cluster Generation
- Computing a representative
  - Average/median vertex position
  - Error quadrics
- Mesh generation
- Topology changes

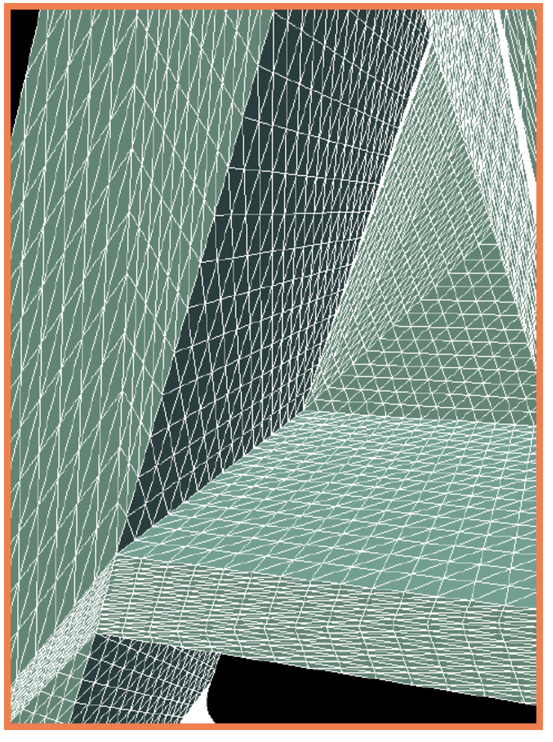
Computing a Representative

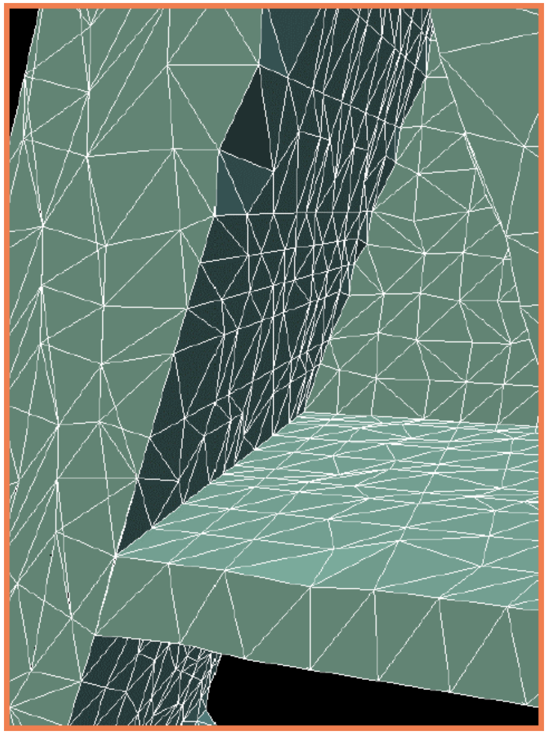




- Average vertex position → Low-pass filter

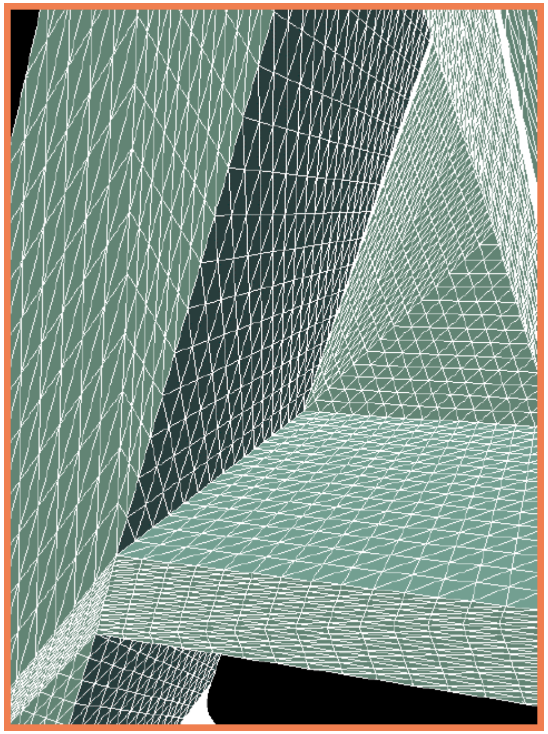
Computing a Representative

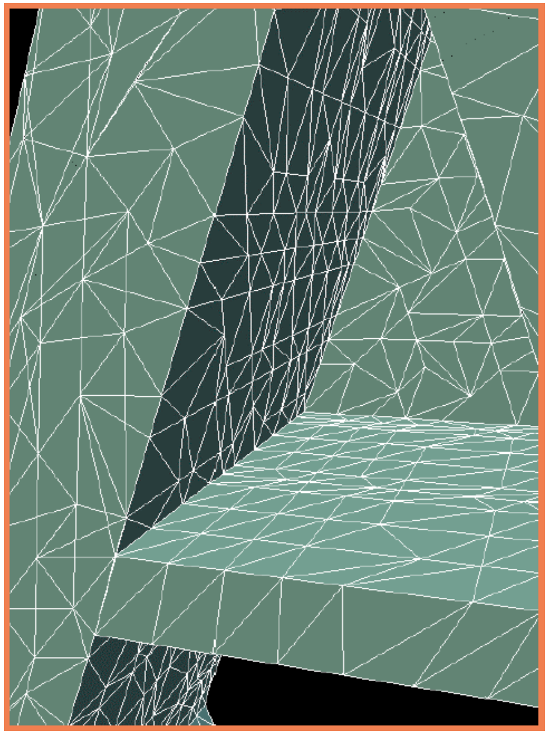




- Median vertex position → Sub-sampling

Computing a Representative





- Error quadrics

Error Quadrics

- Squared distance to plane

$$p = (x, y, z, 1)^T, \quad q = (a, b, c, d)^T$$

$$dist(q, p)^2 = (q^T p)^2$$

$$Q_q = \begin{bmatrix} a^2 & ab & ac & ad \\ ab & b^2 & bc & bd \\ ac & bc & b^2 & cd \\ ad & bd & cd & d^2 \end{bmatrix}$$

Error Quadrics

- Sum distances to vertex' planes

$$\sum_i dist(q_i, p)^2$$

- Point location that minimizes the error

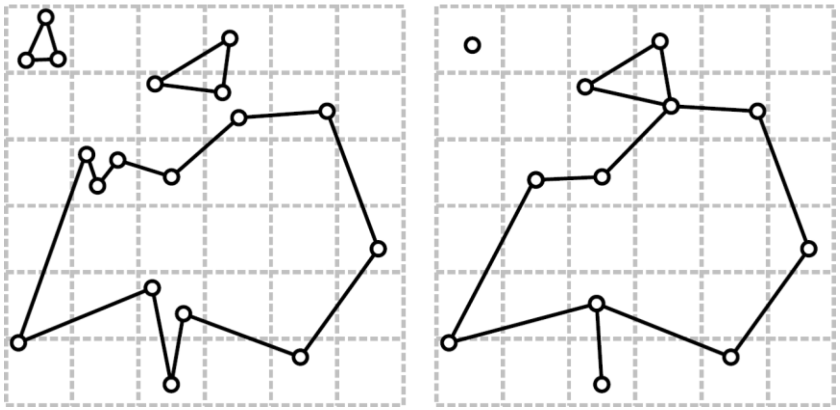
$$\begin{bmatrix} q_{11} & q_{12} & q_{13} & q_{14} \\ q_{21} & q_{22} & q_{23} & q_{24} \\ q_{31} & q_{32} & q_{33} & q_{34} \\ 0 & 0 & 0 & 1 \end{bmatrix} p^* = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

Vertex Clustering

- Cluster Generation
- Computing a representative
- Mesh generation
  - Clusters $p\{p_0,\ldots,p_n\}$, $q\{q_0,\ldots,q_m\}$
  - Connect (p,q) if there was an edge (p_i,q_j)
- Topology changes

Vertex Clustering

- Cluster Generation
- Computing a representative
- Mesh generation
- Topology changes
  - If different sheets pass through one cell
  - Not manifold



Simplification

- Vertex Clustering
- **Iterative Decimation**
- Extensions

Iterative Decimation

- General Setup
- Decimation operators
- Error metrics
- Fairness criteria
- Topology changes

General Setup

Repeat:

- pick mesh region
- apply decimation operator

Until no further reduction possible

Greedy Optimization

For each region
- evaluate quality after decimation
- enqueue(quality, region)

Repeat:
- pick best mesh region
- apply decimation operator
- update queue

Until no further reduction possible

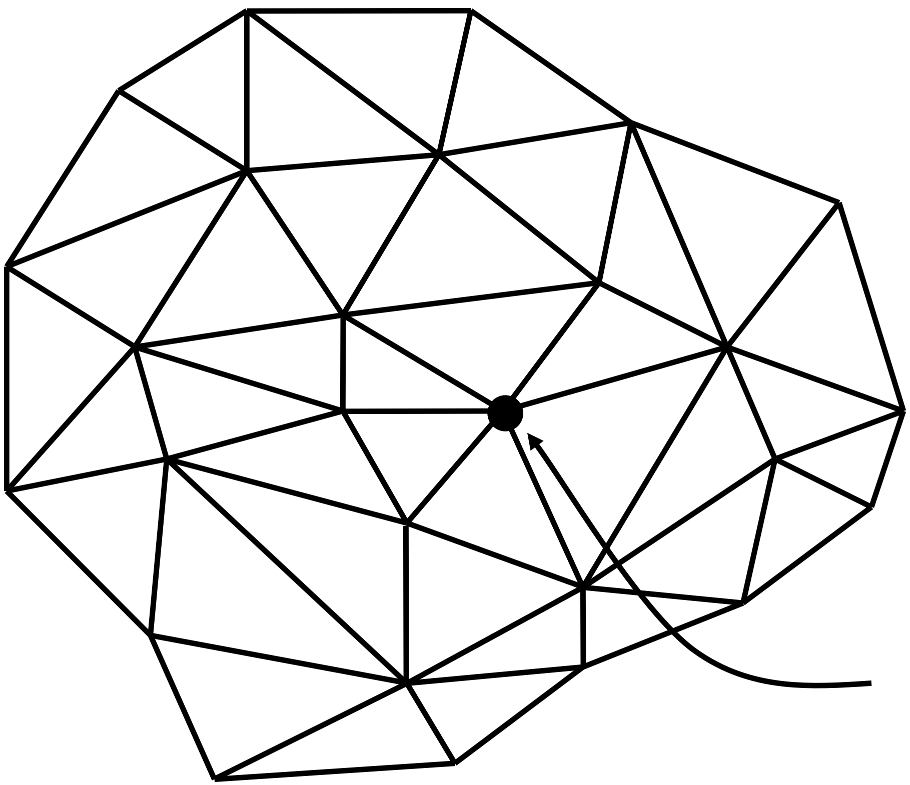
Iterative Decimation

- General Setup
- Decimation operators
- Error metrics
- Fairness criteria
- Topology changes

Decimation Operators

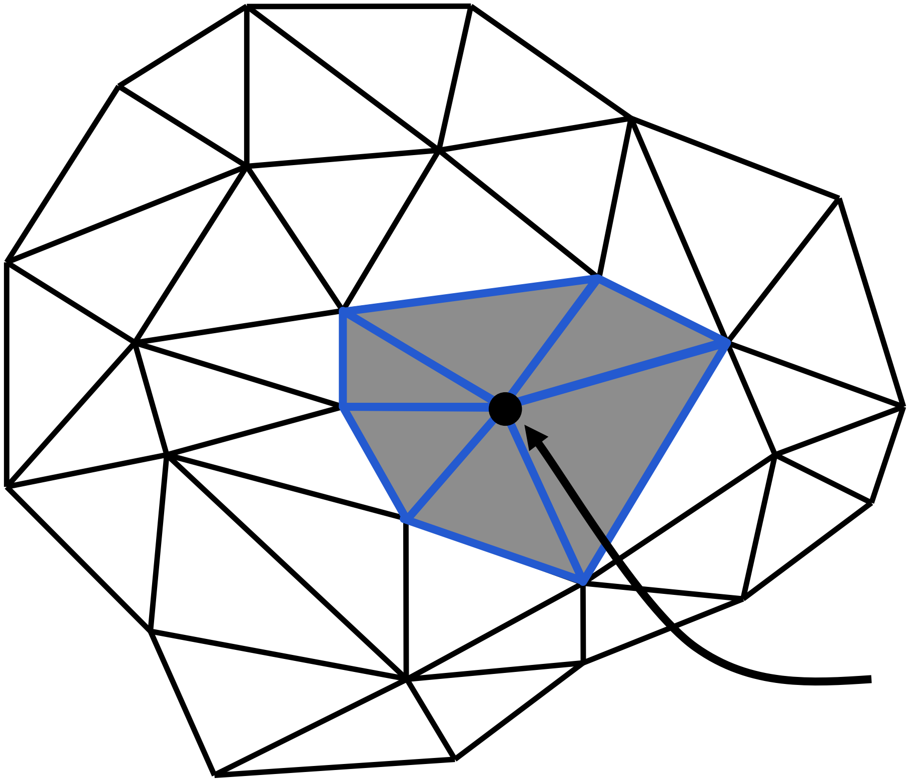
- What is a "region" ?
- What are the DOF for re-triangulation?
- Classification
  - Topology-changing vs. topology-preserving
  - Subsampling vs. filtering

Vertex Removal



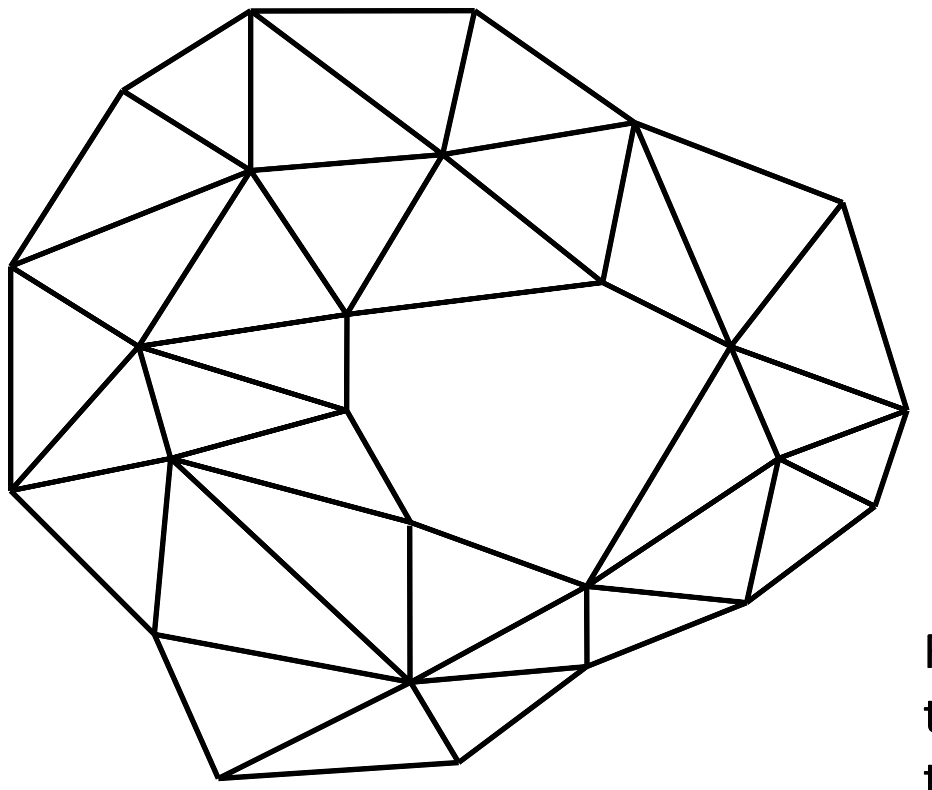
Select a vertex to be eliminated

Vertex Removal



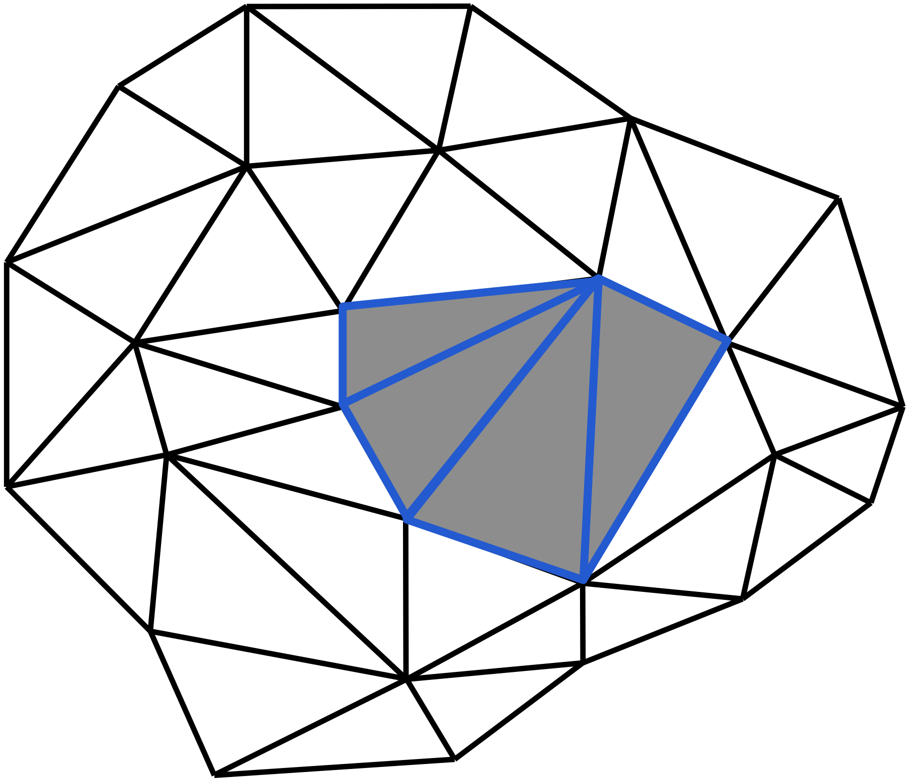
Select all triangles
sharing this vertex

Vertex Removal



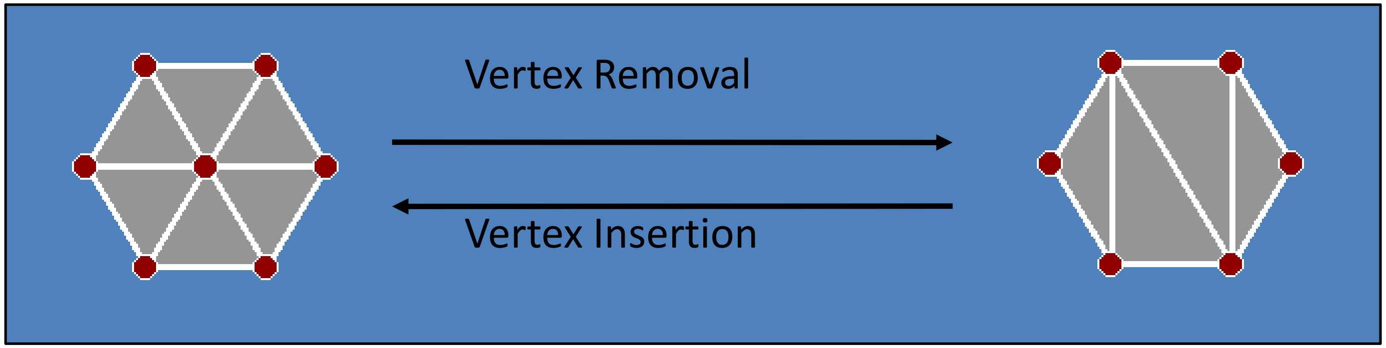
Remove the selected triangles, creating the hole

Vertex Removal



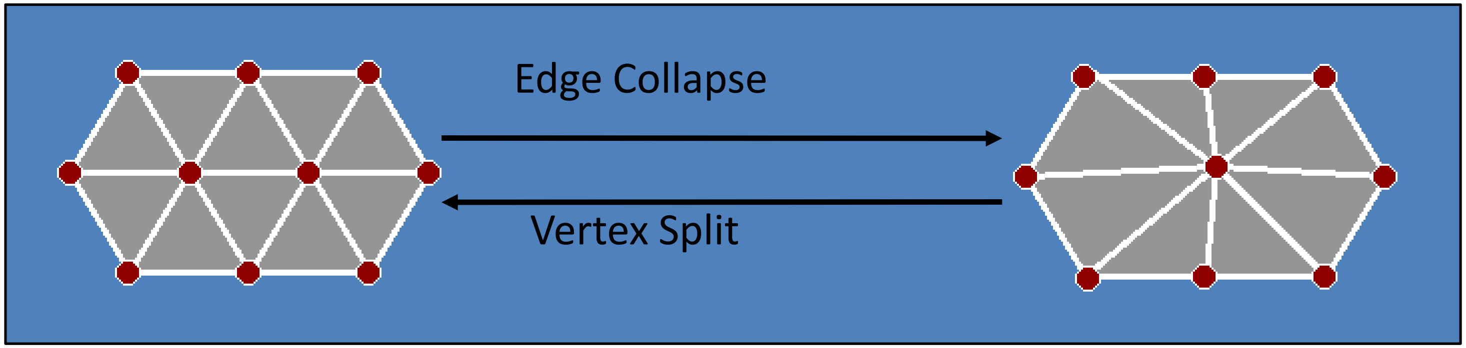
Fill the hole with triangles

Decimation Operators



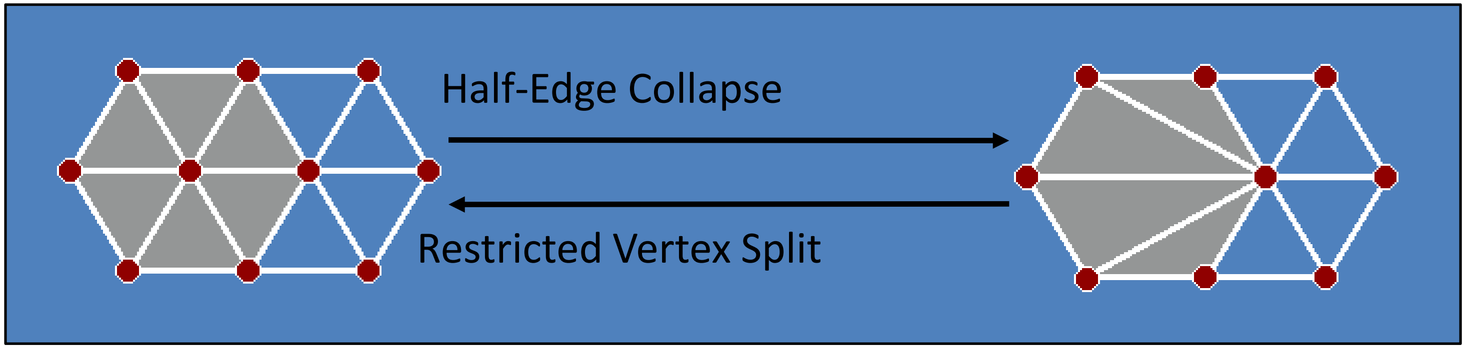
- Remove vertex
- Re-triangulate hole
  - Combinatorial DOFs
  - Sub-sampling

Decimation Operators



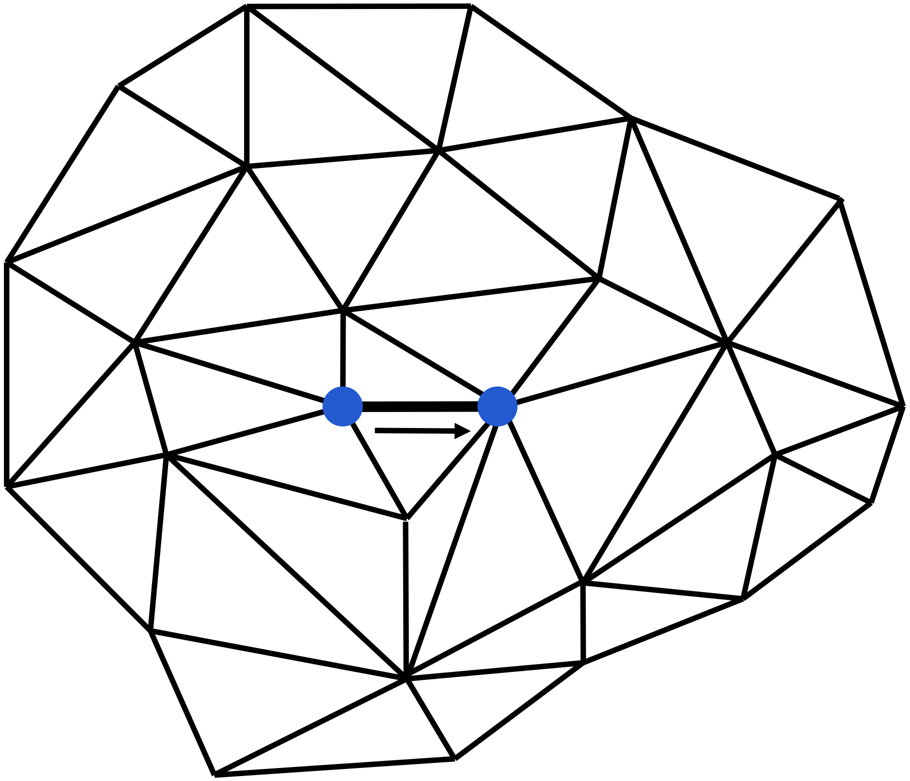
- Merge two adjacent triangles
- Define new vertex position
  - Continuous DOF
  - Filtering

Decimation Operators

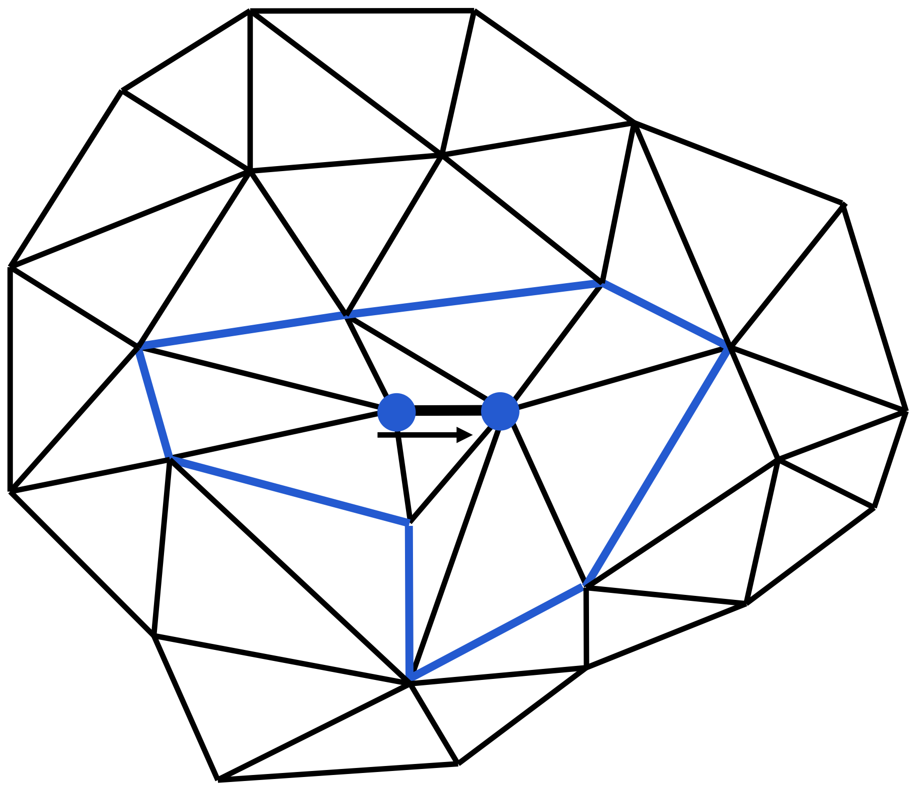


- Collapse edge into one end point
  - Special vertex removal
  - Special edge collapse
- No DOFs
  - One operator per half-edge
  - Sub-sampling

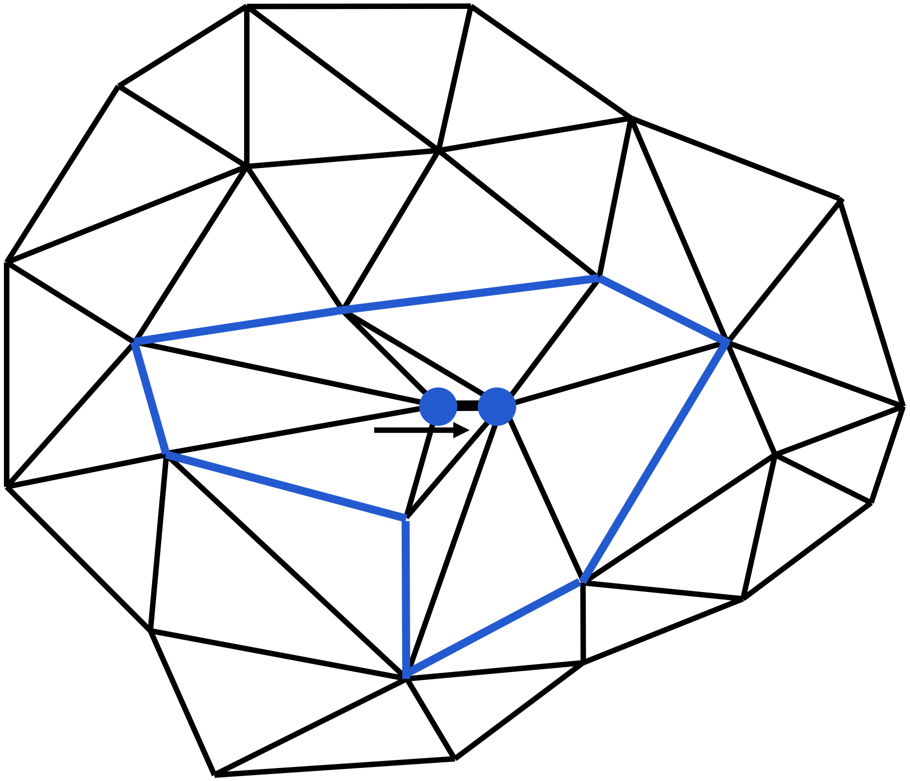
Edge Collapse



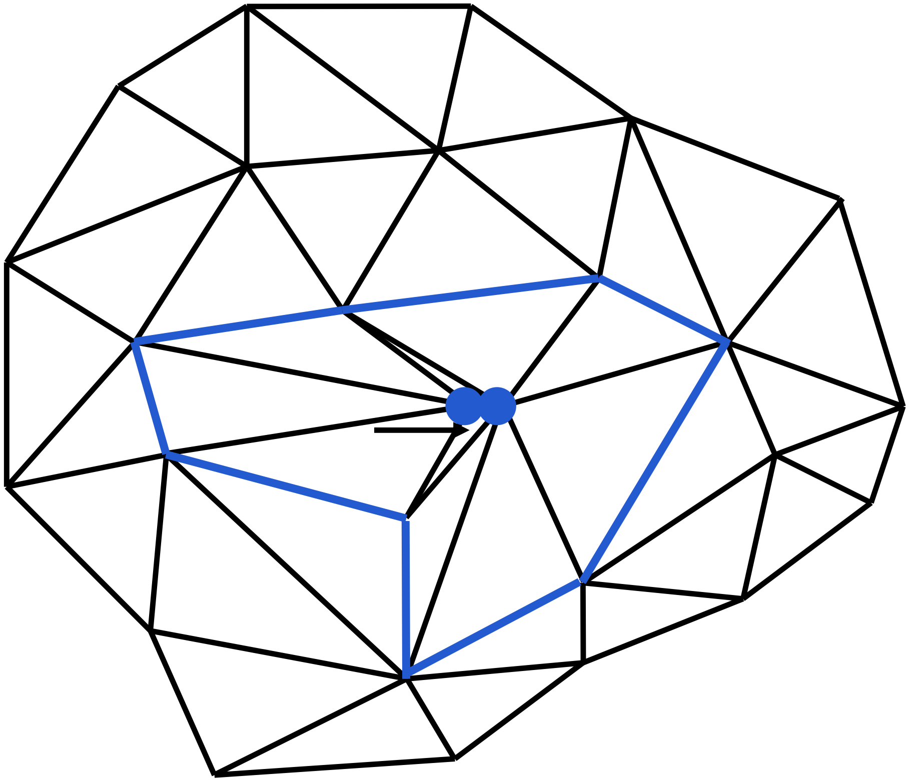
Edge Collapse



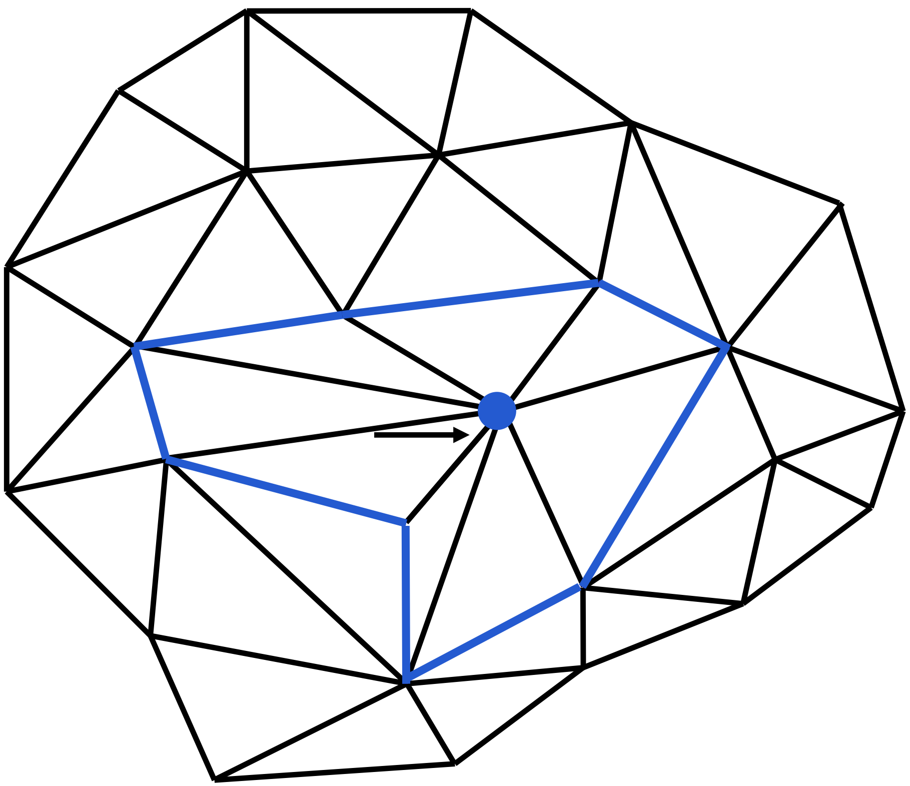
Edge Collapse



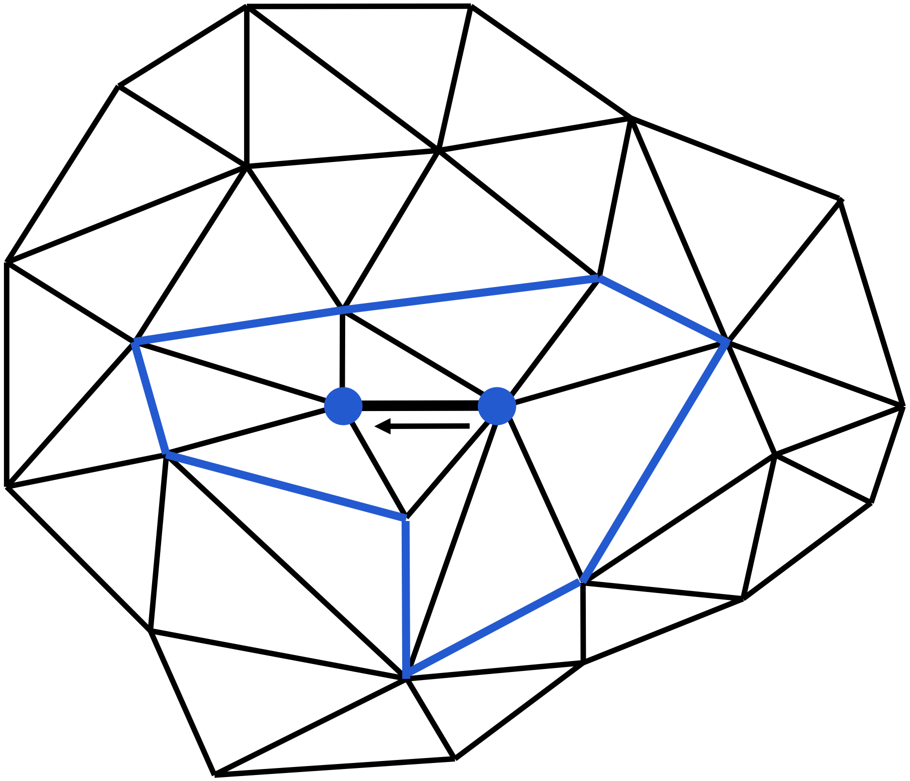
Edge Collapse



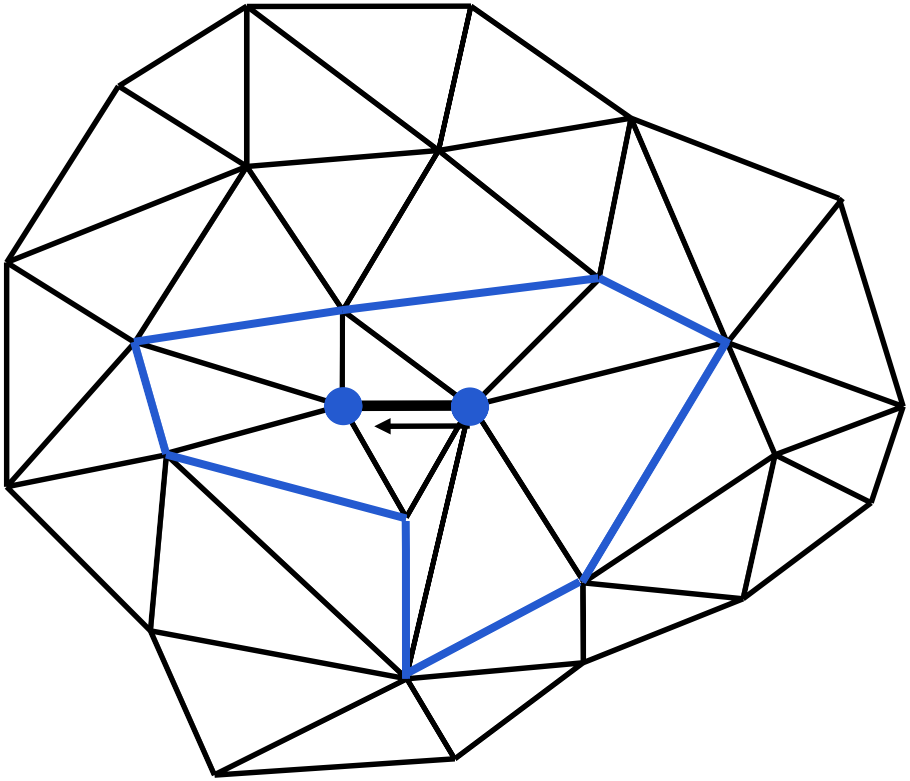
Edge Collapse



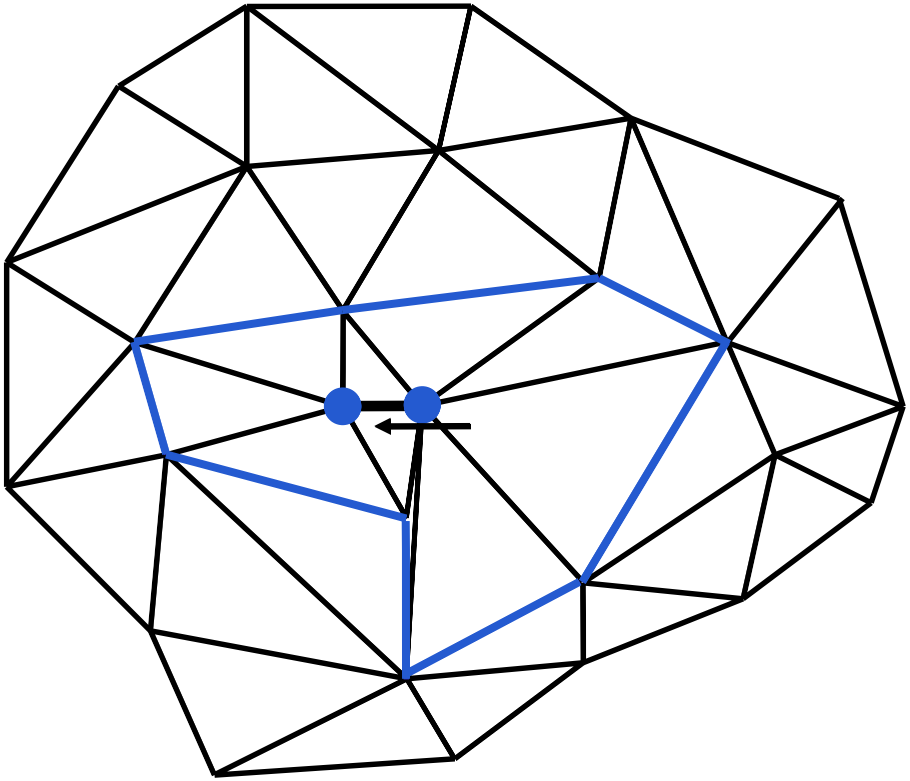
Edge Collapse



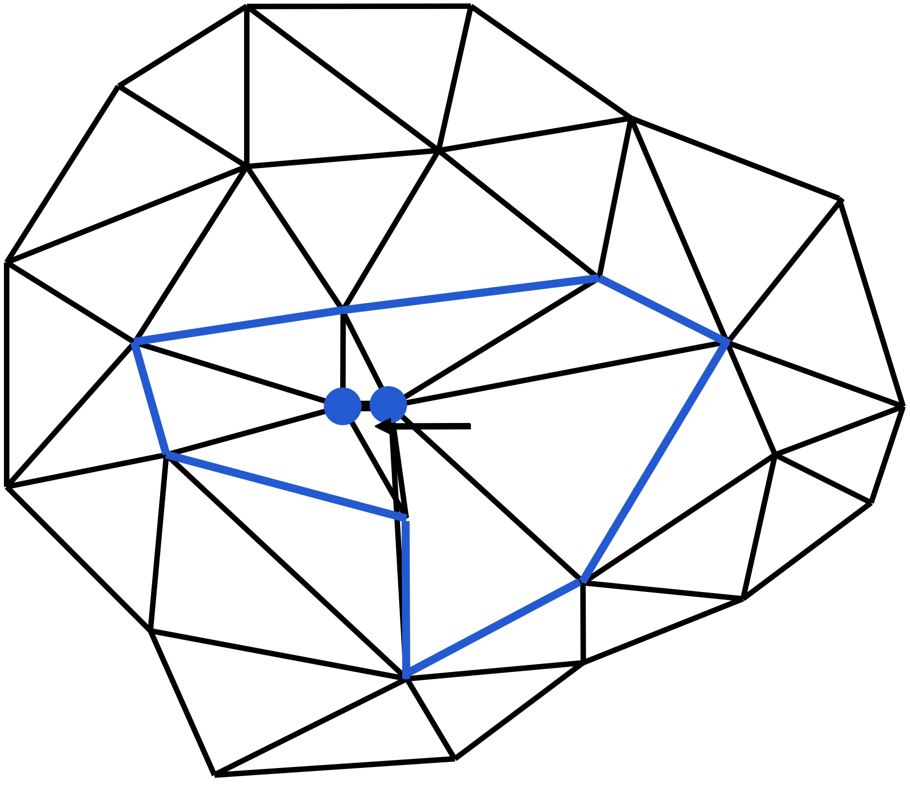
Edge Collapse



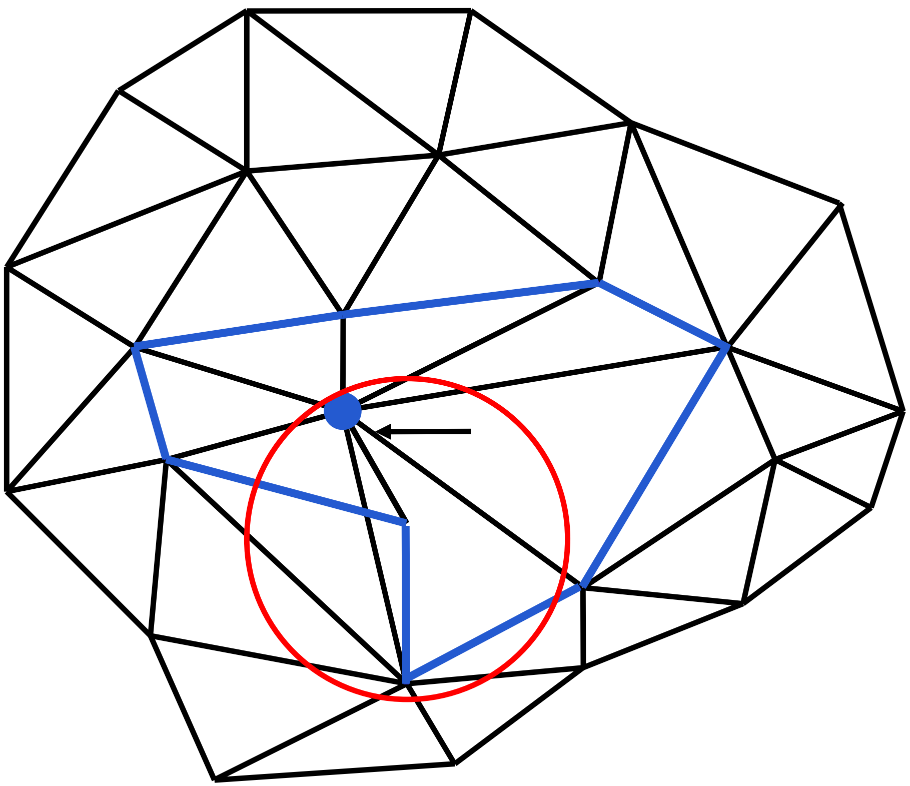
Edge Collapse



Edge Collapse



Edge Collapse

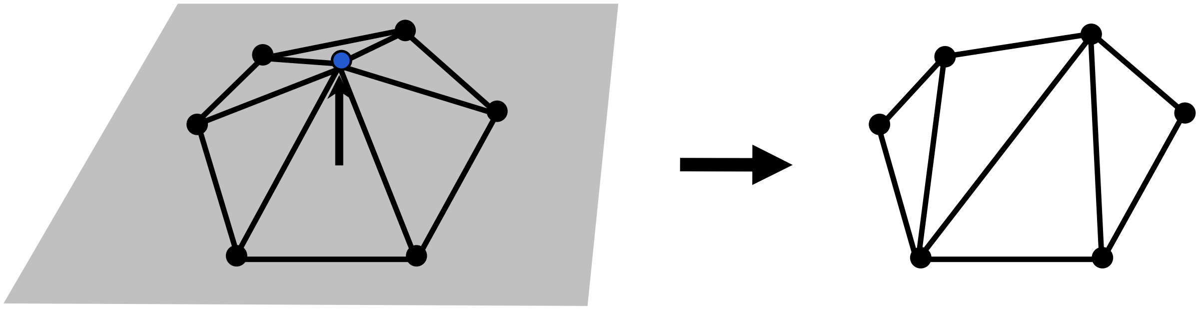


Incremental Decimation

- General Setup
- Decimation operators
- Error metrics
- Fairness criteria
- Topology changes

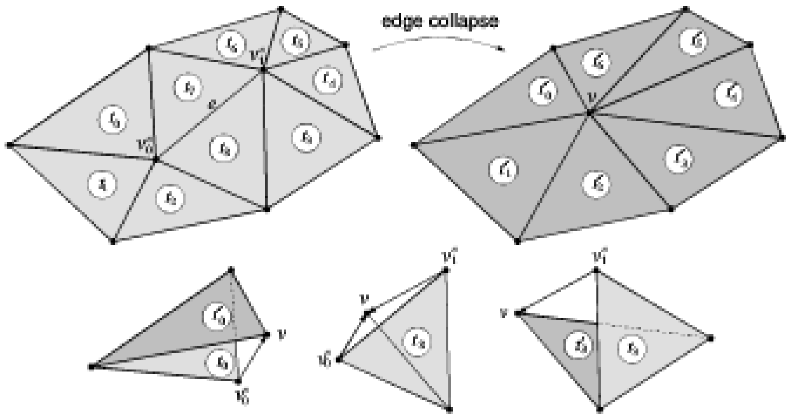
Local Error Metrics

- Local distance to mesh [Schroeder et al. 92]
  - Compute average plane
  - No comparison to *original* geometry



Local Error Metrics

- Volume preserving [Lindstrom-Turk]. *Fast and memory efficient polygonal simplification*. *IEEE Visualization* 98.

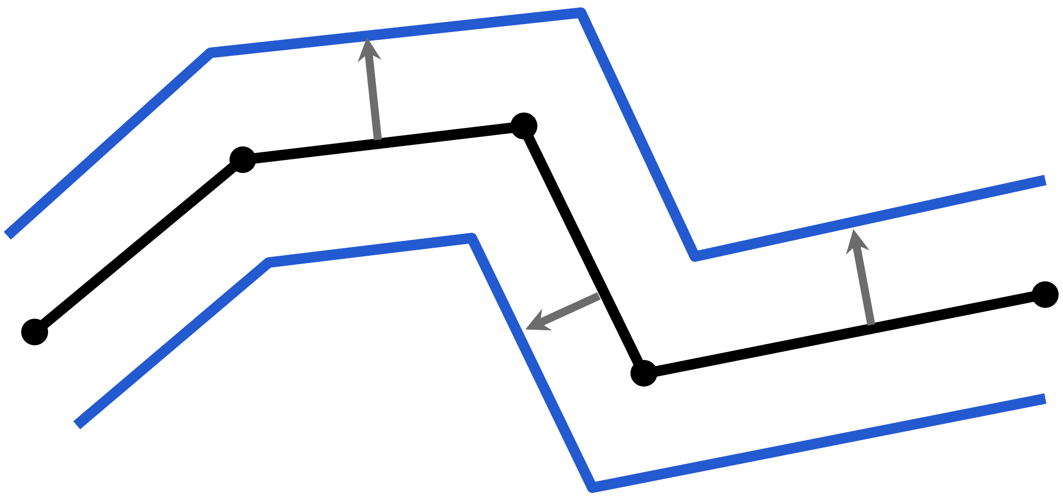


Implemented in

CGAL

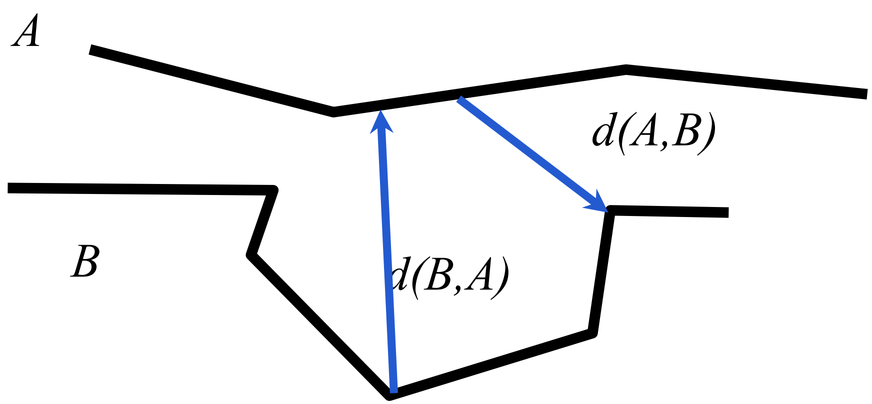
Global Error Metrics

- Simplification envelopes [Cohen et al. 96]
  - Compute (non-intersecting) offset surfaces
  - Simplification guarantees to stay within bounds



Global Error Metrics

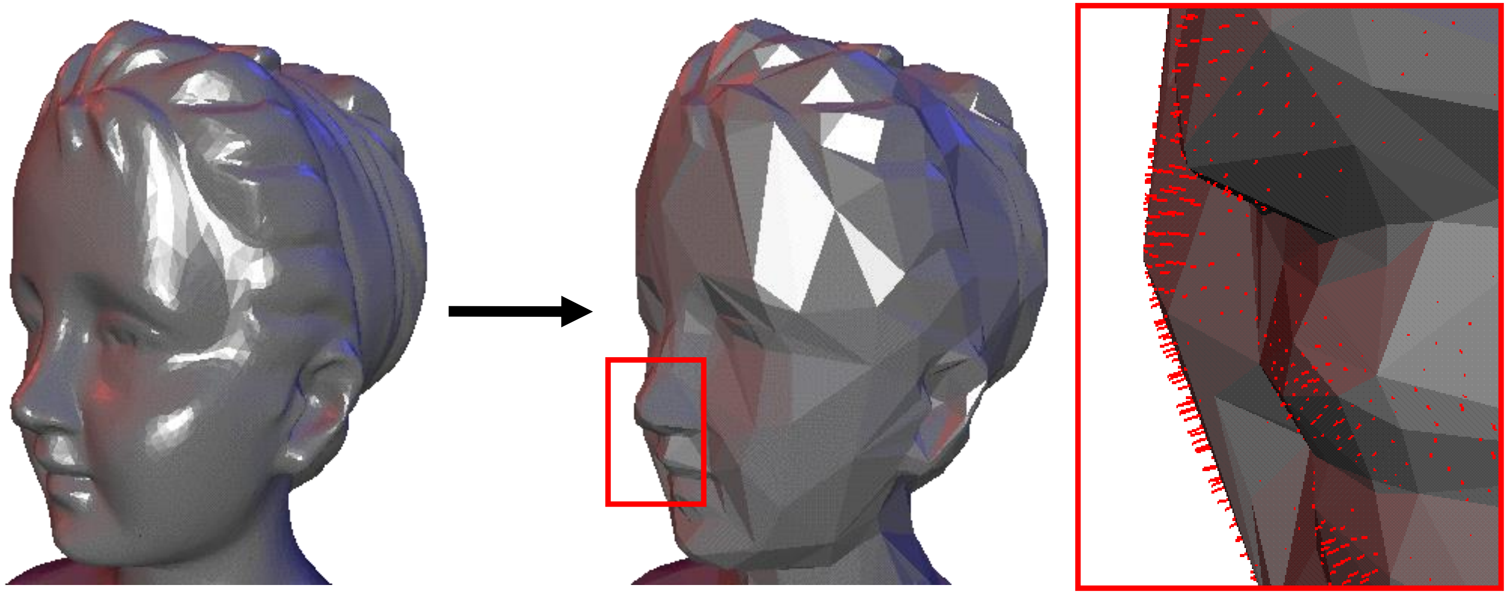
- (Two-sided) Hausdorff distance: Maximum distance between two shapes
  - In general $d(A,B) \neq d(B,A)$
  - Compute-intensive



Valette et al. *Mesh Simplification using a two-sided error minimization*. 2012.

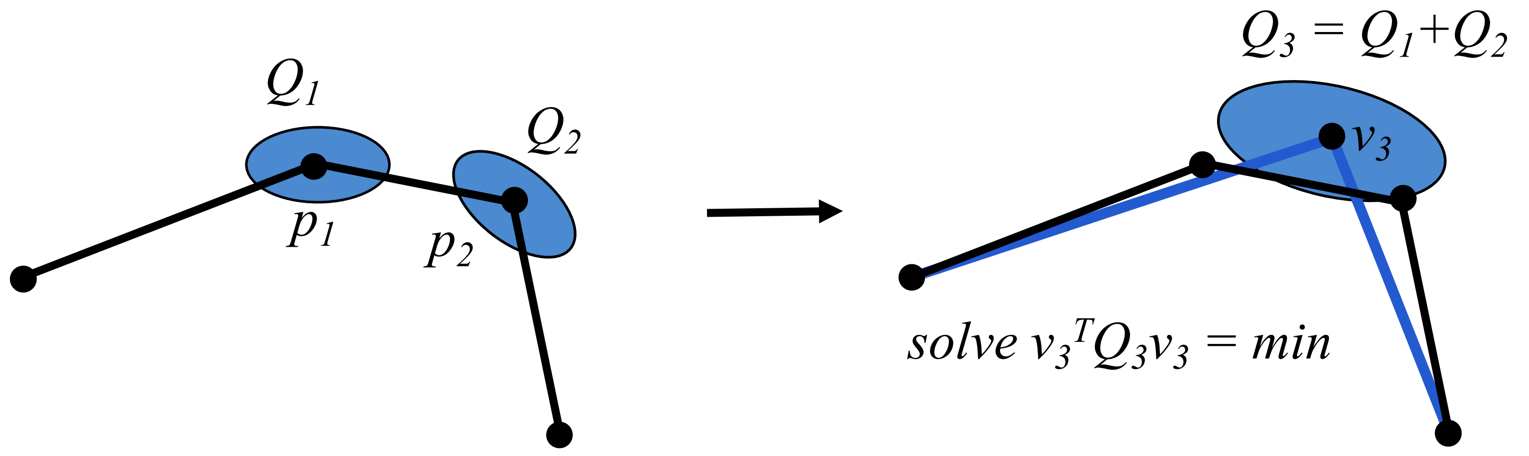
Global Error Metrics

- One-sided Hausdorff distance
  - From original vertices to current surface

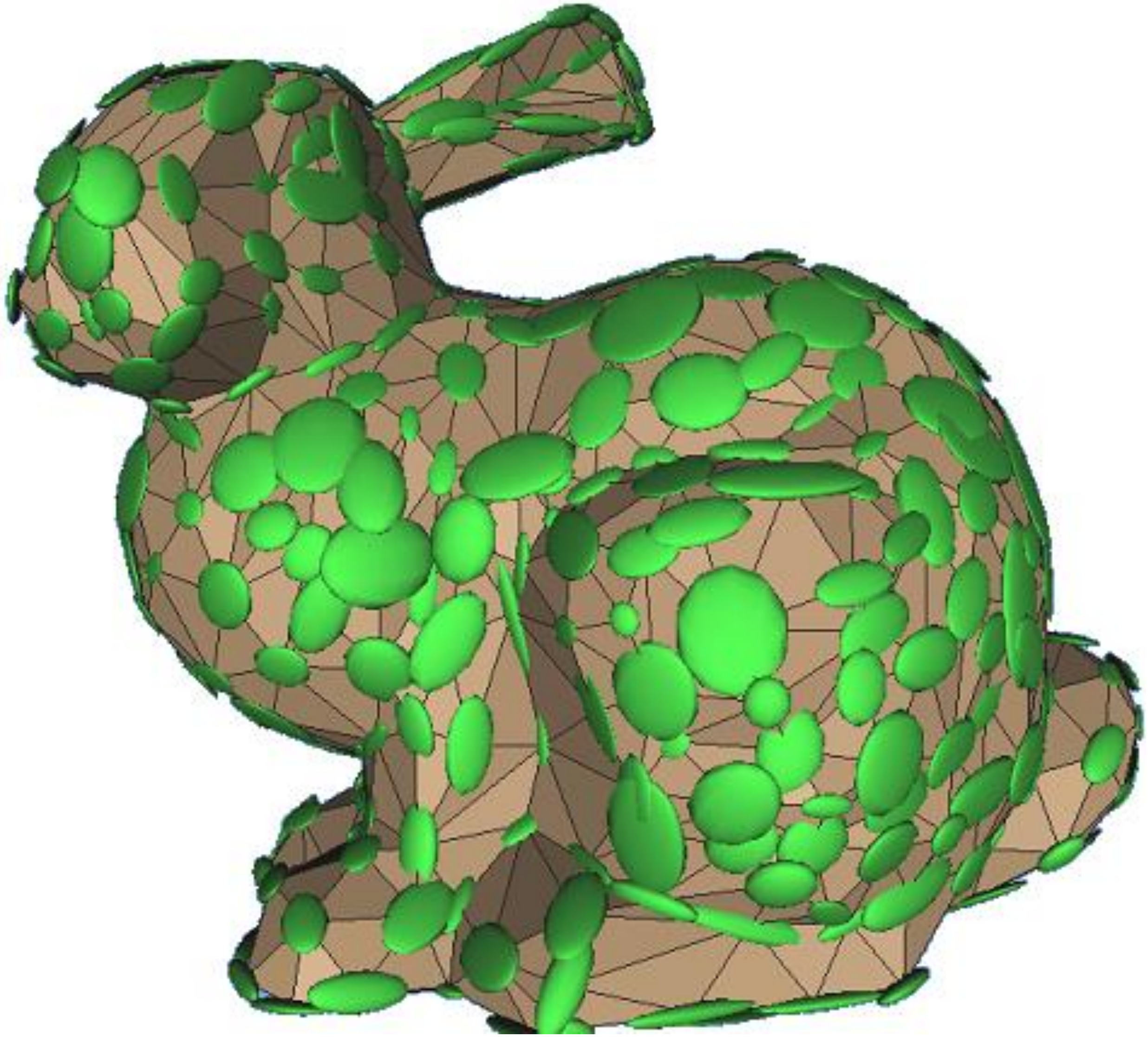


Global Error Metrics

- Error quadrics [Garland, Heckbert 97]
  - Squared distance to planes at vertex
  - No bound on true error



Error Quadrics

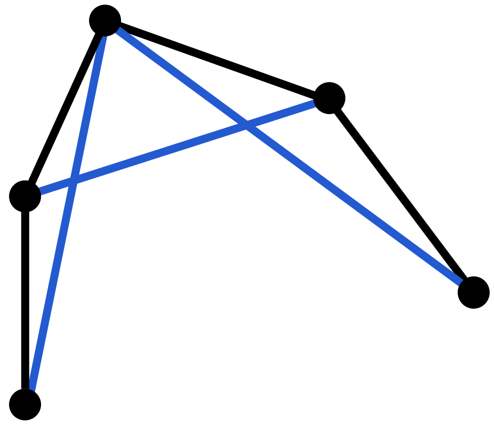


Incremental Decimation

- General Setup
- Decimation operators
- Error metrics
- Fairness criteria
- Topology changes

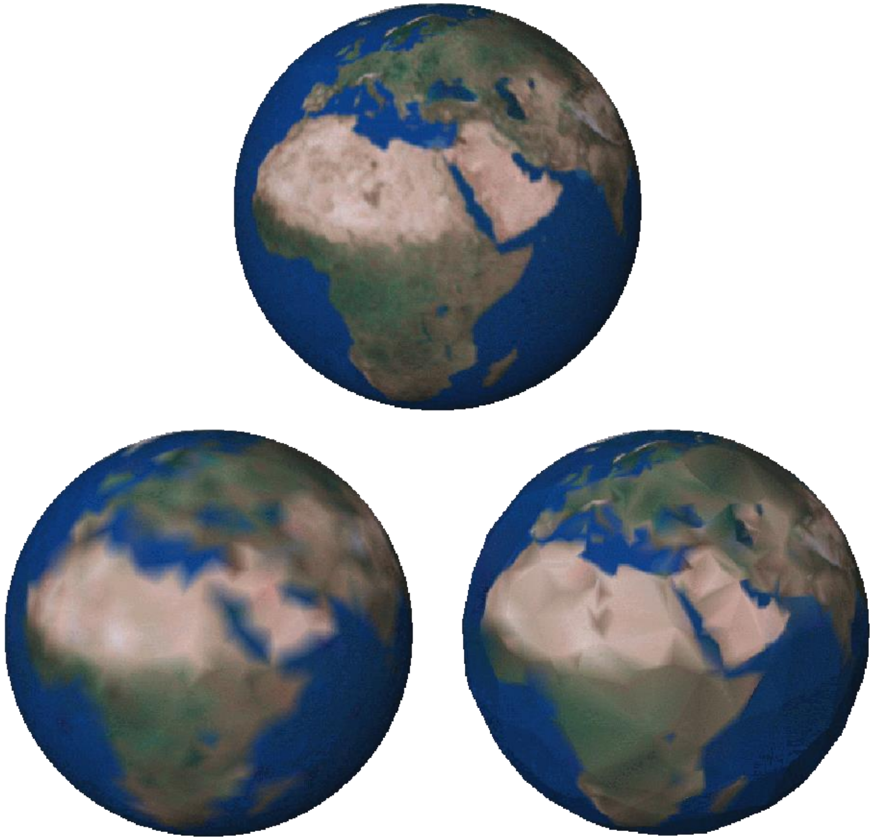
Fairness Criteria

- Rate quality of decimation operation
  - Approximation error
  - Triangle shape
  - Dihedral angles
  - Valence balance
  - Color differences
  - ...



Fairness Criteria

- Rate quality after decimation
  - Approximation error
  - Triangle shape
  - Dihedral angles
  - Valance balance
  - Color differences
  - ...

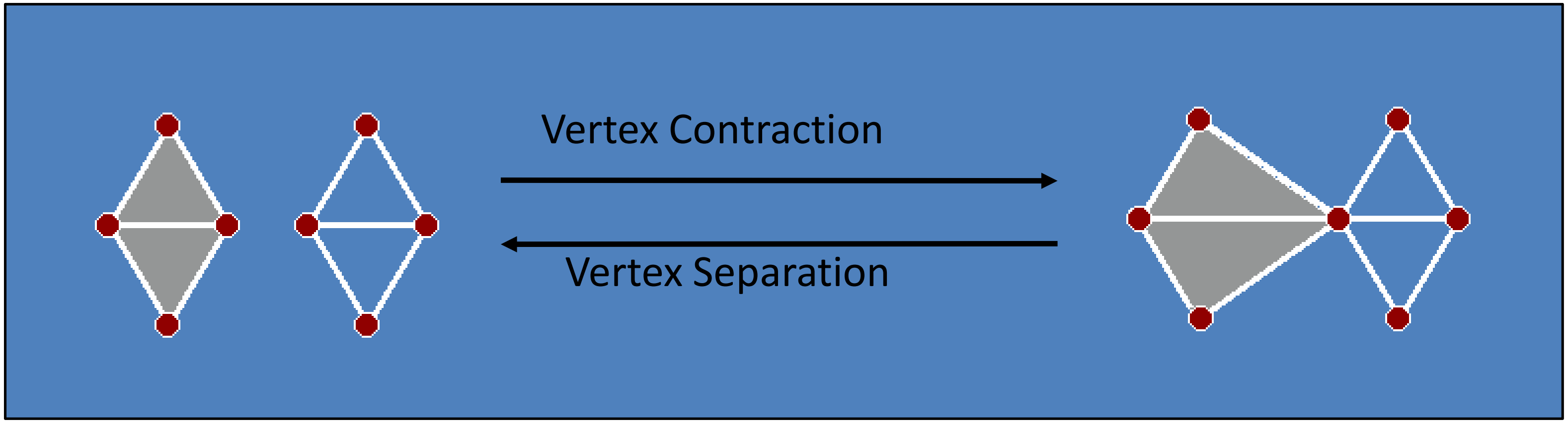


Incremental Decimation

- General Setup
- Decimation operators
- Error metrics
- Fairness criteria
- Topology changes

Topology Changes

- Merge vertices across non-edges
  - Changes mesh topology
  - Need *spatial neighborhood* information
  - Generates *non-manifold* meshes



Topology Changes

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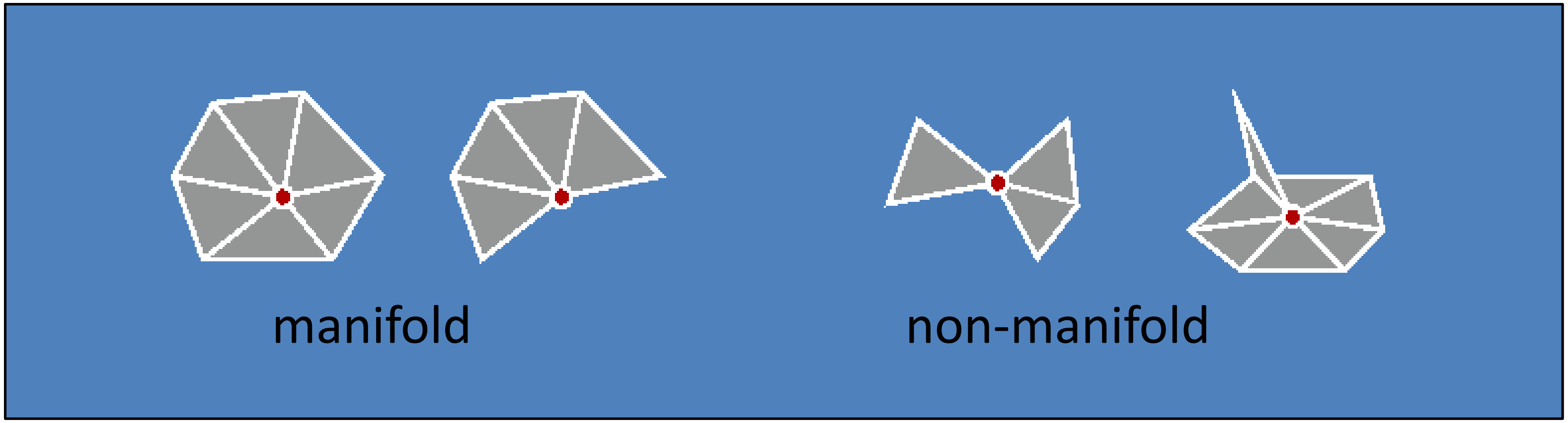


Approximation

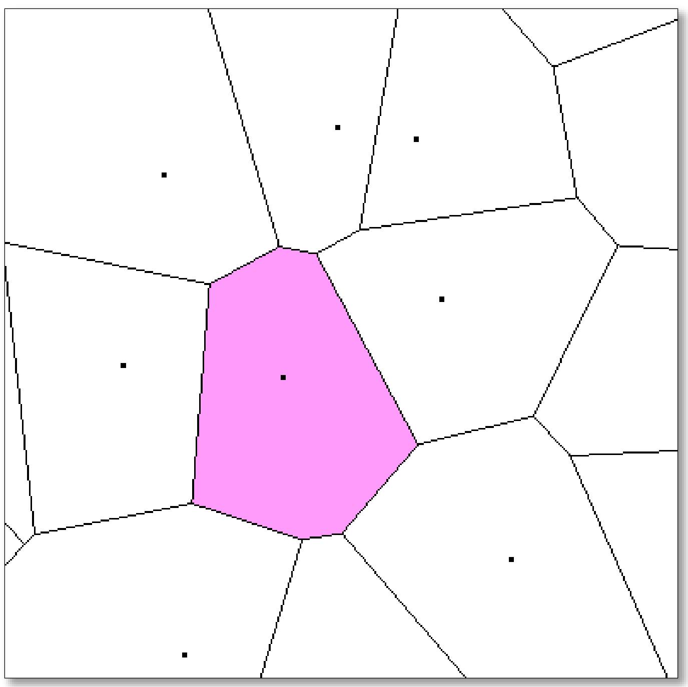
Variational Shape Approximation

- Rationale: cast surface approximation as a variational k-partitioning problem

Cohen-Steiner, A., Desbrun. *Variational Shape Approximation*. SIGGRAPH 2004.

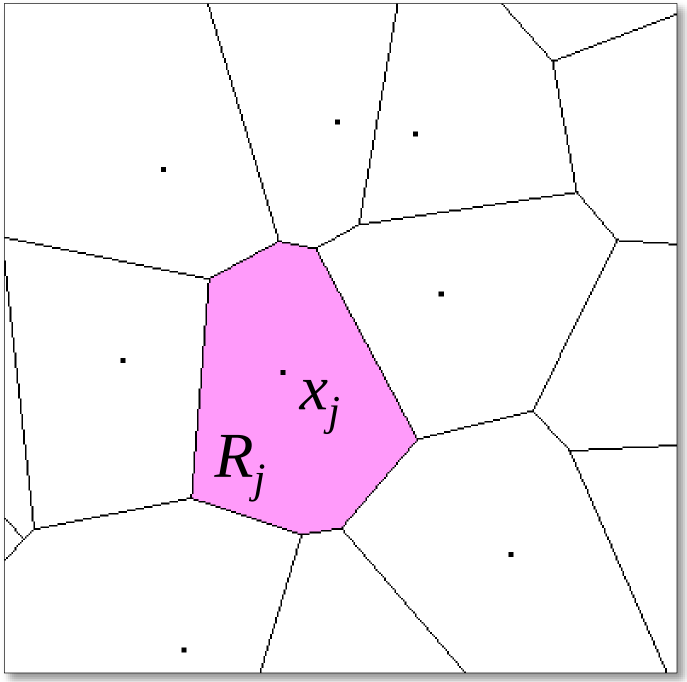
SIGGRAPH2004

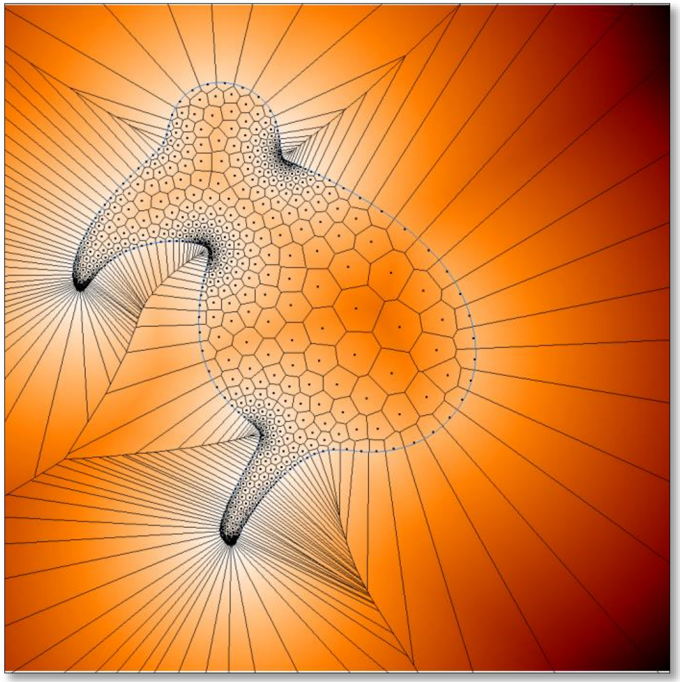
Simpler Setting: 2D Partitioning



Energy

$$E = \sum_{j=1..k} \int_{x \in R_j} \rho(x) \| x - x_j \|^2 \, dx$$

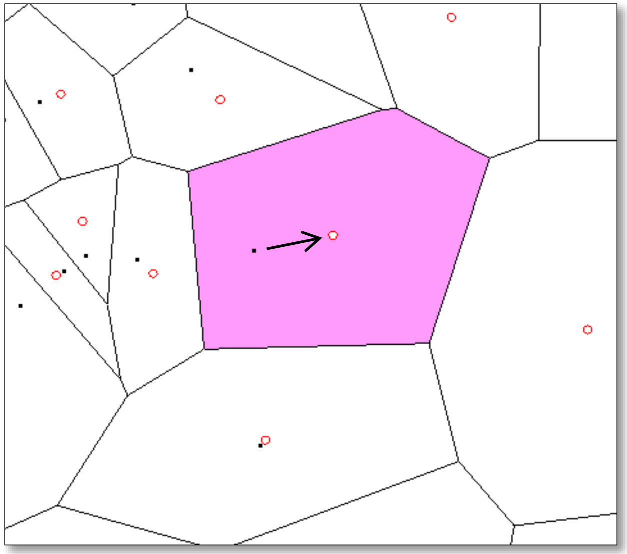




density function

Lloyd Iteration

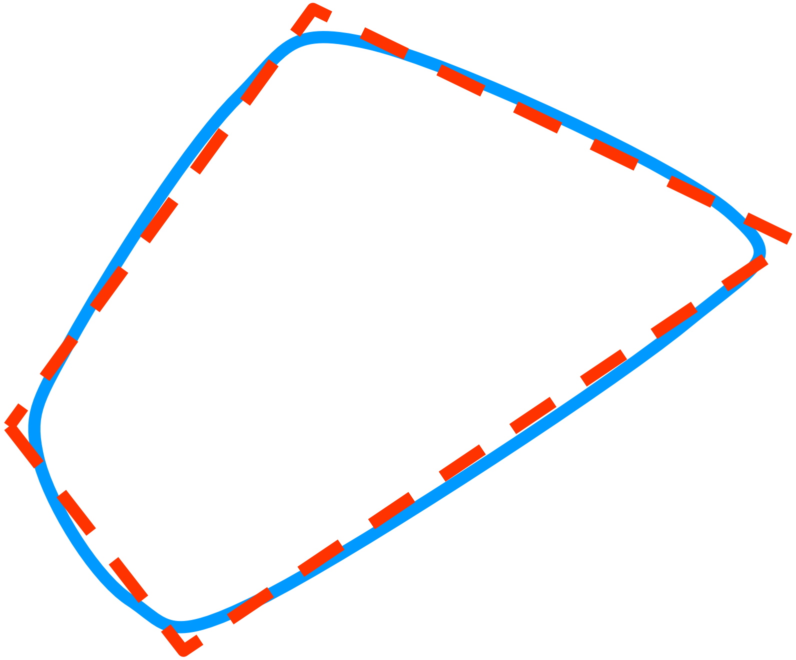
- Alternate:
  - Voronoi partitioning
  - Relocate sites to centroids
- Minimizes energy
  - Necessary condition for optimality: Centroidal Voronoi tessellation



demo

Variational Shape Approximation

- Rationale: cast surface approximation as a variational k-partitioning problem
  - for each region, find best-fit *linear proxy*
    - "best fit" for a given metric



demo

Variational Shape Approximation

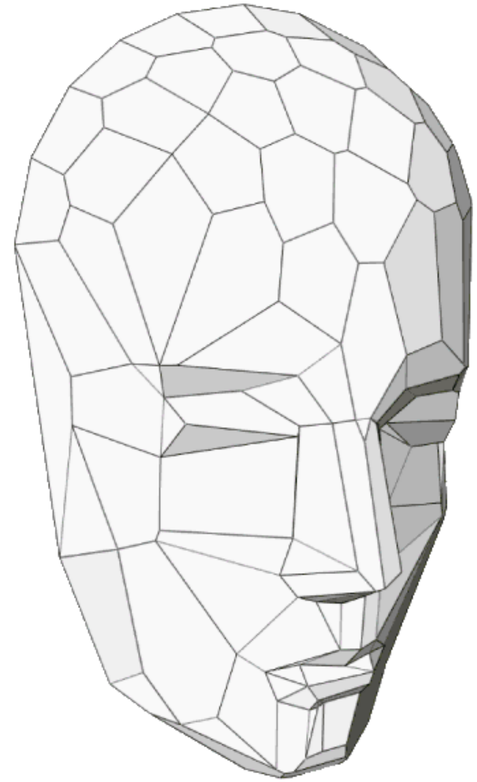
- Distortion

  = integrated error between region and proxy
- Total distortion = sum of proxy distortion
- Best k-approximation = minimum distortion

Overview

initial mesh
+ partition

associated
proxies



proxy-based
remeshing

K-Means Clustering

Starting with k-generators

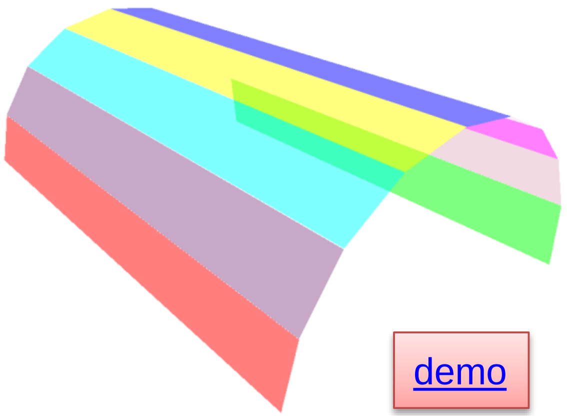
Alternate:

- cluster by closest proximity (creates regions R_j)
- find new generators c_j of regions R_j

Partition Optimization

Clustering for Approximation

- Replace points by proxies
- Min approximation error
- Equi-distribute energy among proxies

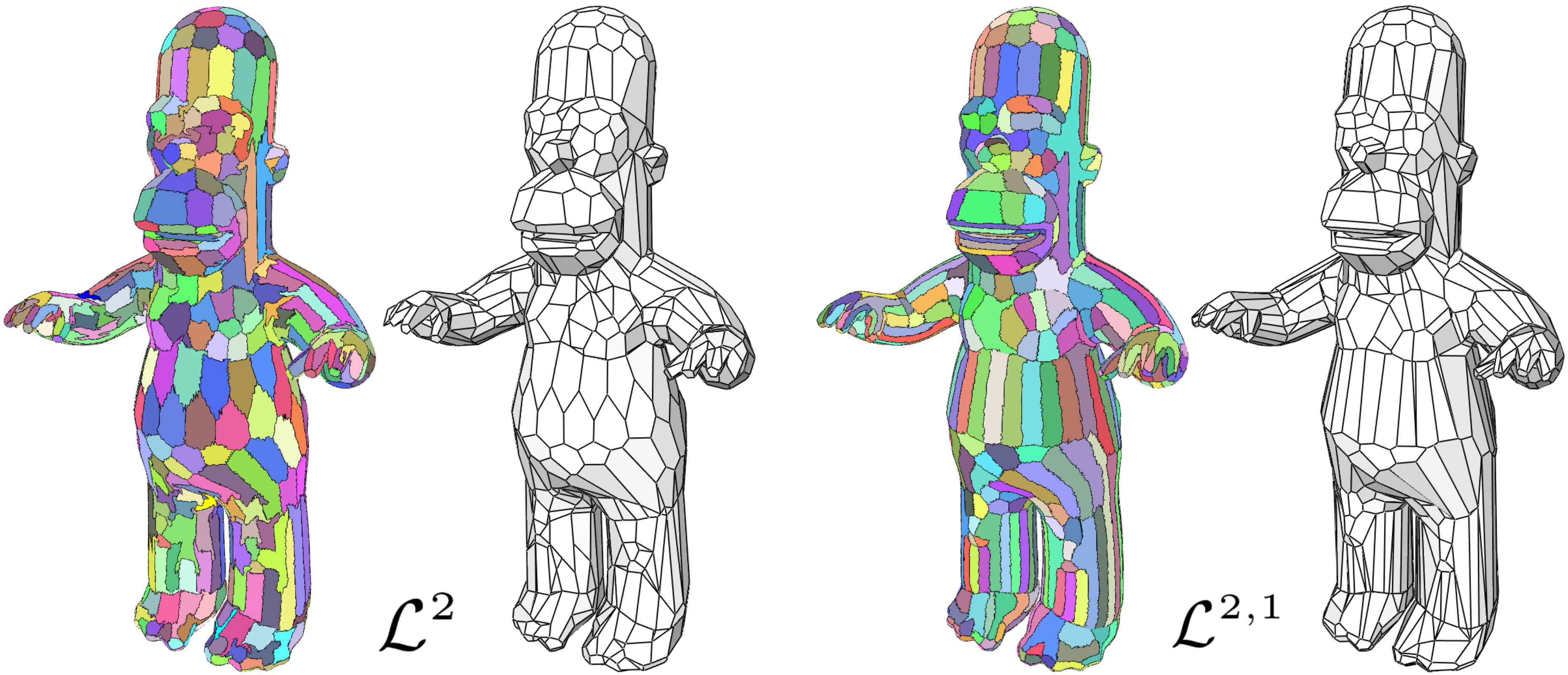


demo

Error Metrics

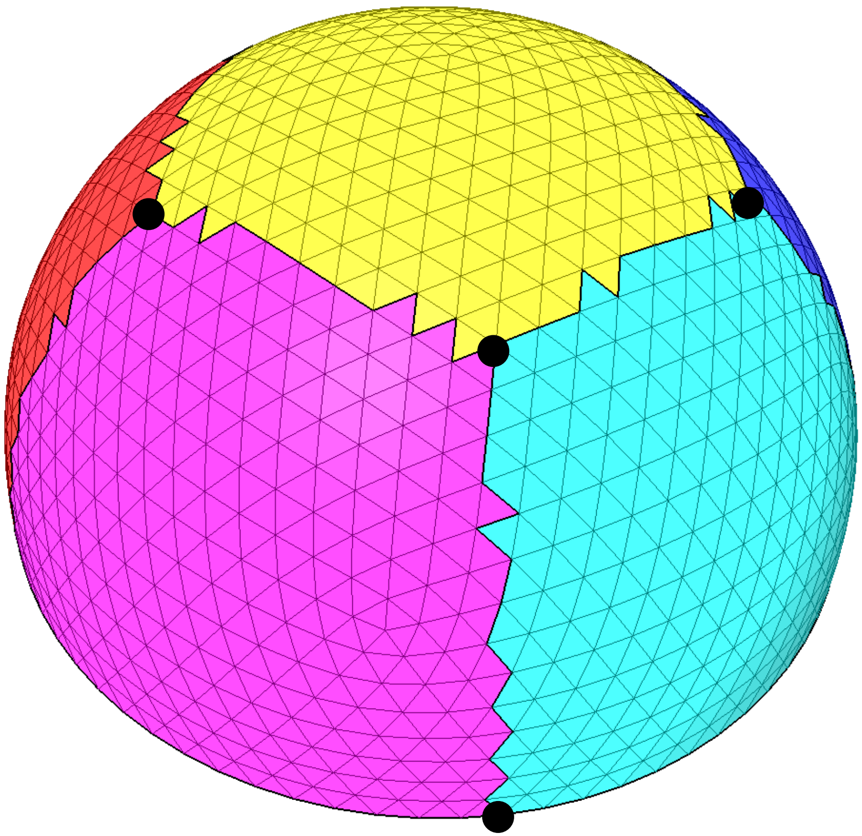
- L^2
  - asymptotically, aspect ratio is $\sqrt{\kappa_1/\kappa_2}$
  - hyperbolic regions troublesome
    - no unique minimum
  - convergence in L^2 does not guarantee in normals
    - example: Schwarz's Chinese lantern
    - [Shewchuck 04] gradient bounds harder than interpolation
- $L^{2,1}$ $\iint_{x\in X} \|\mathbf{n}(x) - \mathbf{n}_i\|^2 dx$
  - asymptotically, aspect ratio is κ_1/κ_2
  - hyperbolic regions ok
  - captures normal field

L^2 vs. $L^{2,1}$



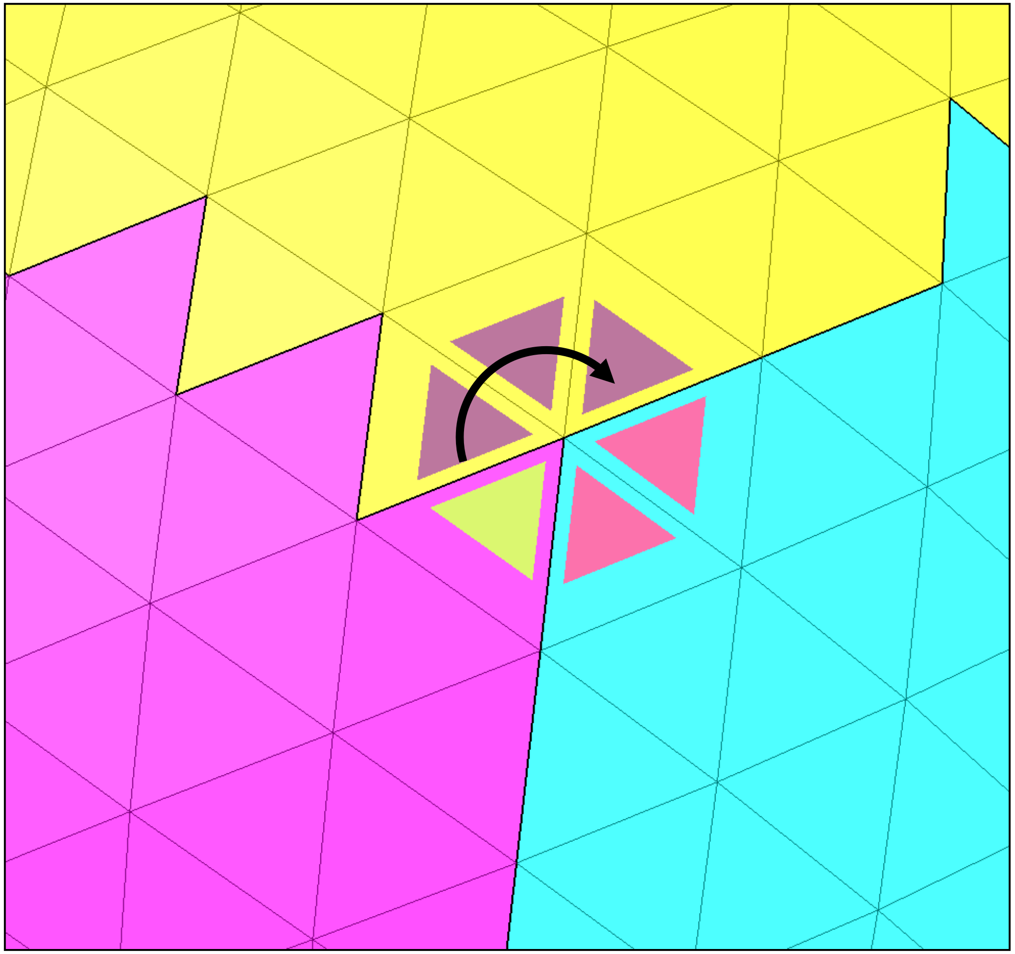
Triangulation

- node vertex
  - where 3+ regions meet
  - 2+ on boundary



Triangulation

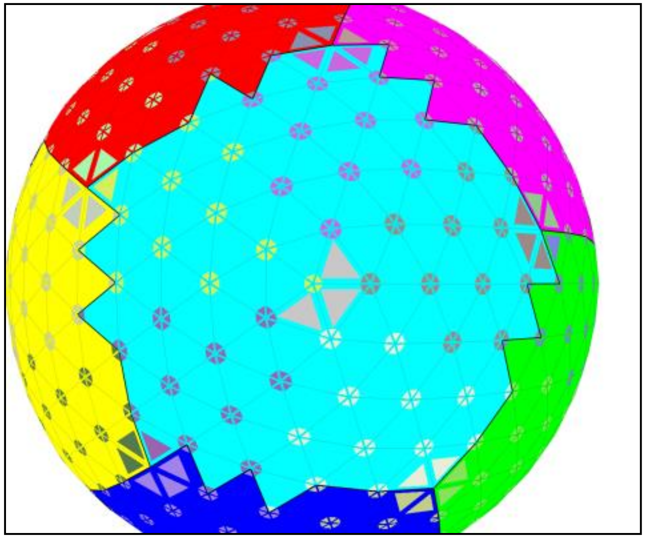
- node wedge



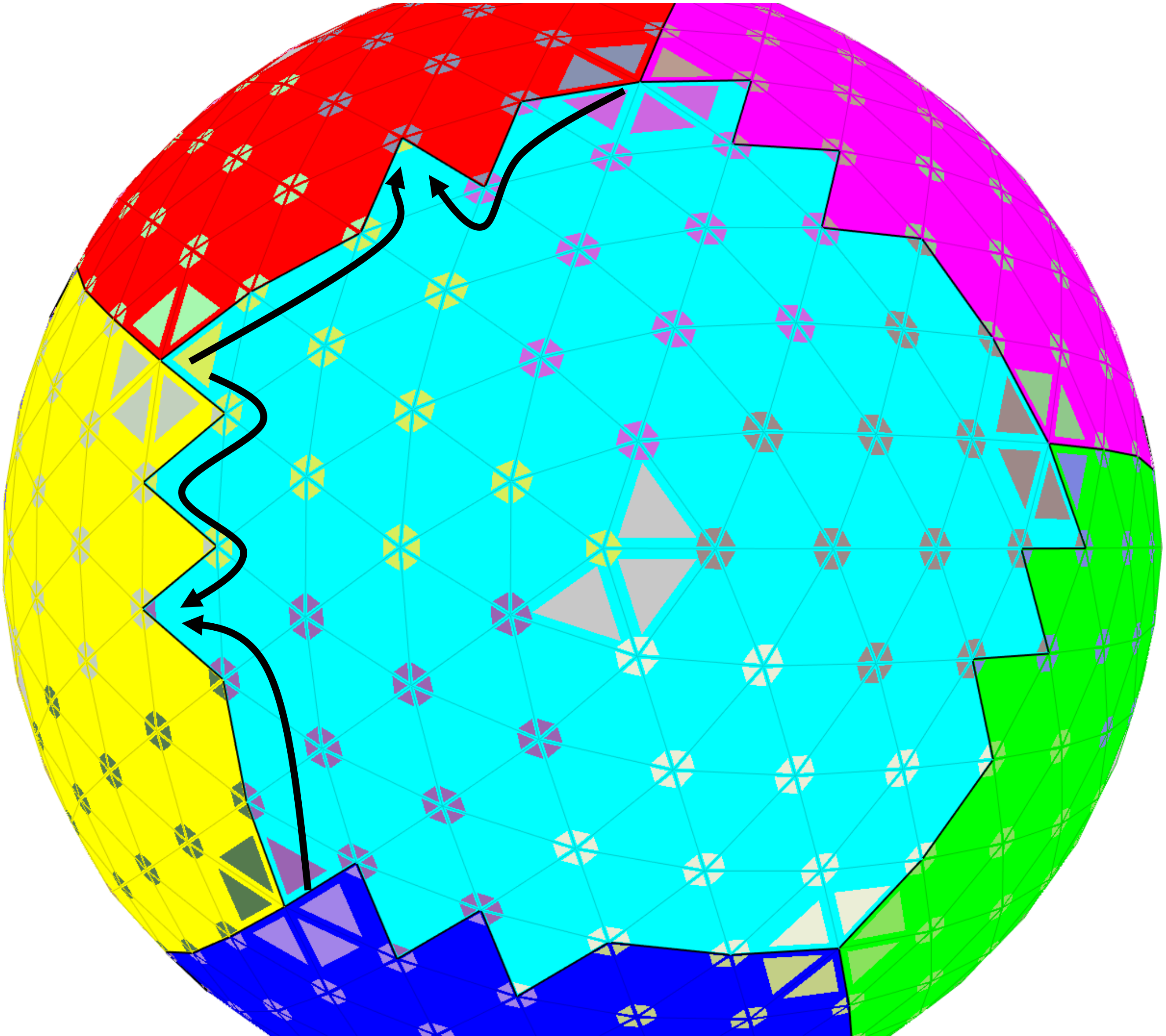
Triangulation

- Two-pass flooding algorithm (~multi-source Djisktra's shortest path algorithm)

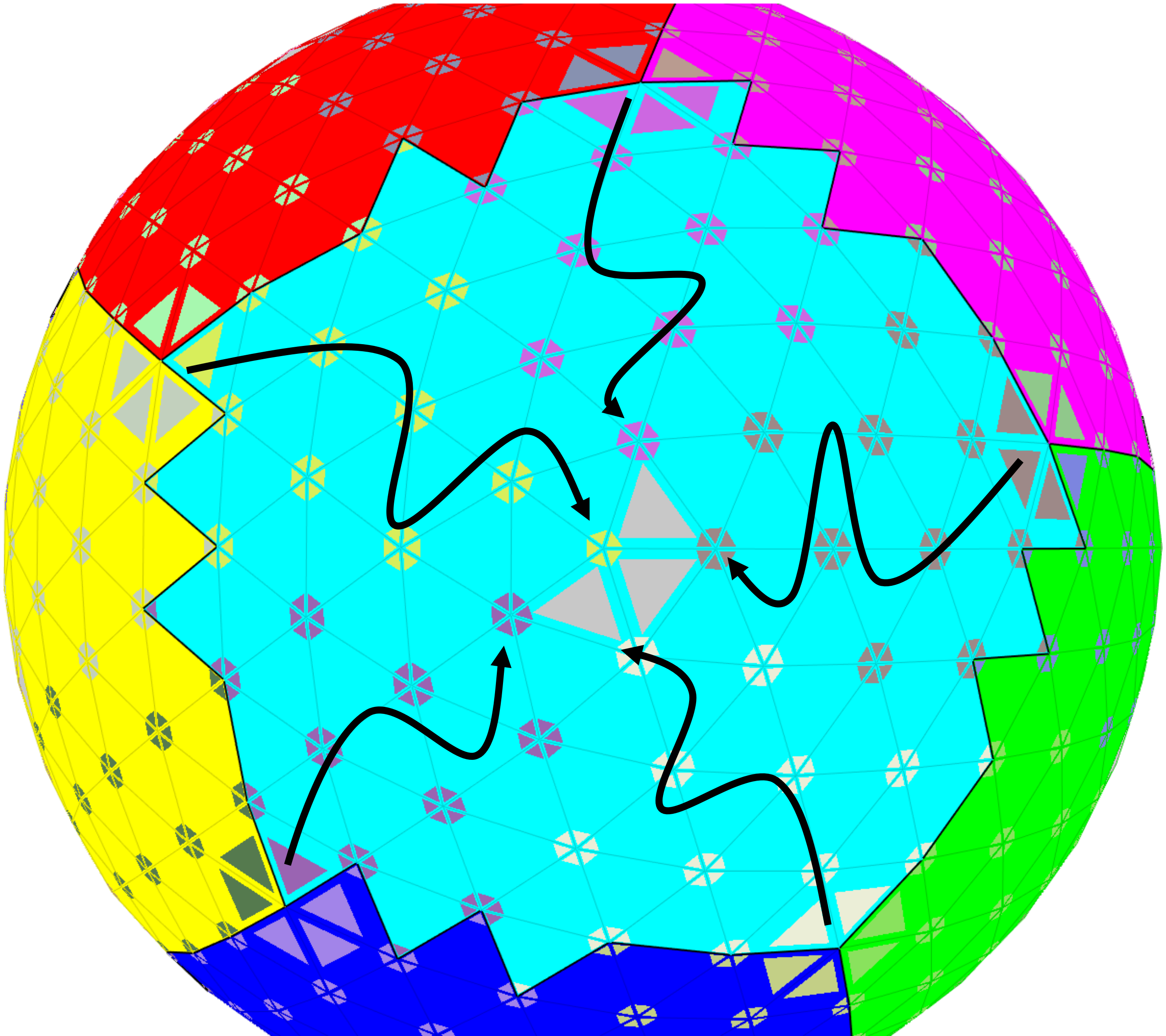
- first pass: flood only region boundaries (to enforce the *constrained edges)*
- second pass: flood interior areas



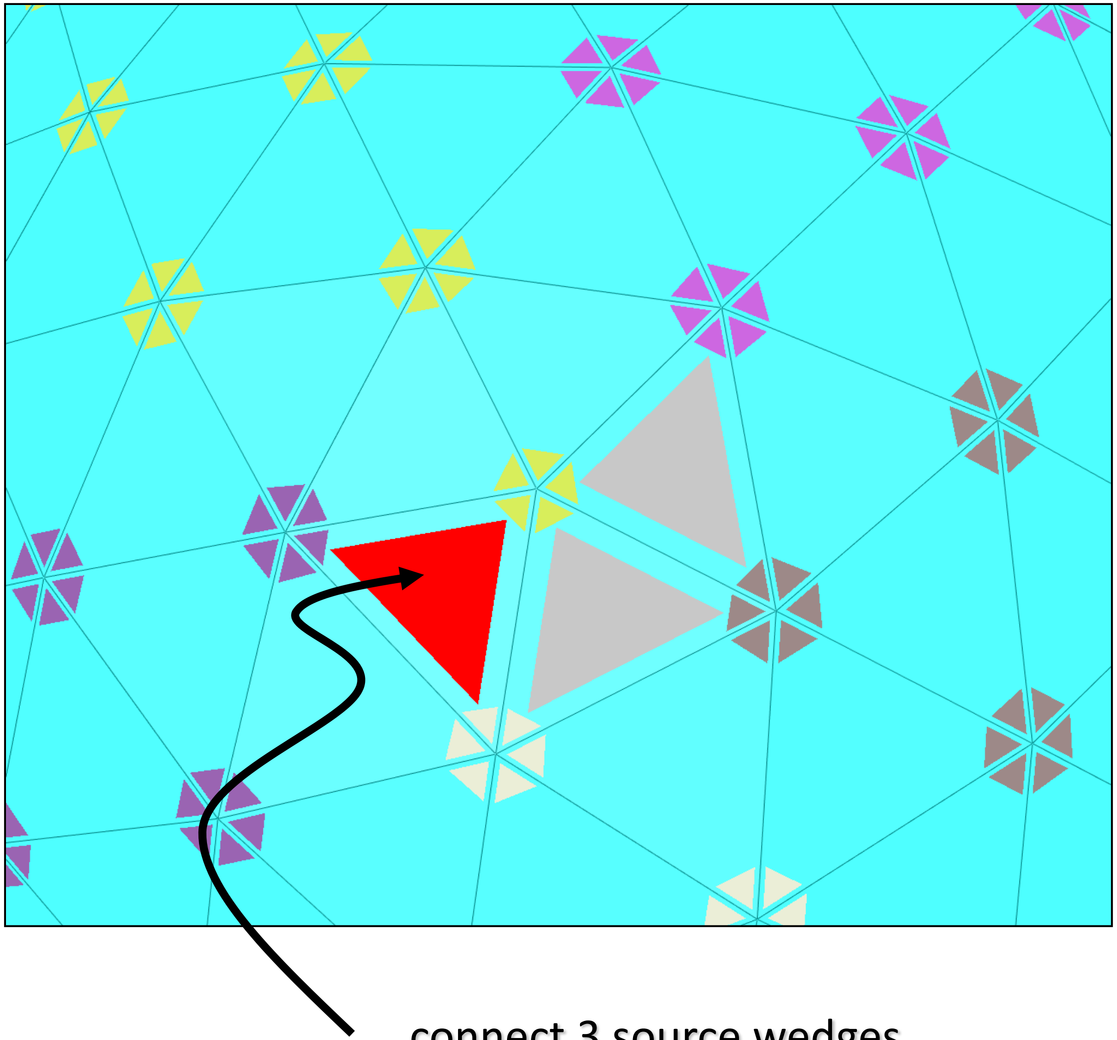
First pass



Second pass

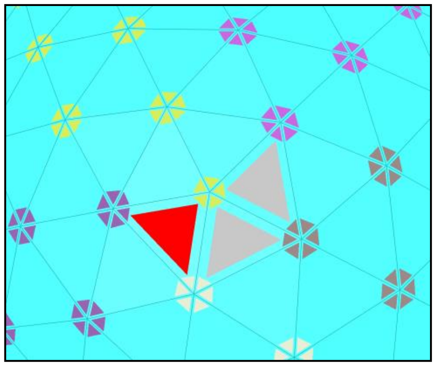


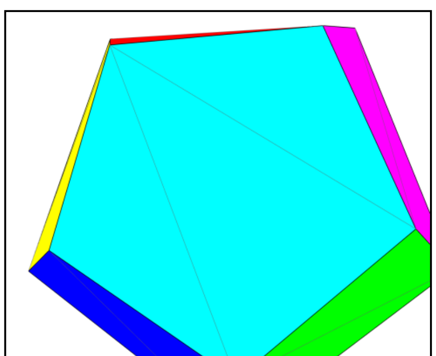
Triangulation

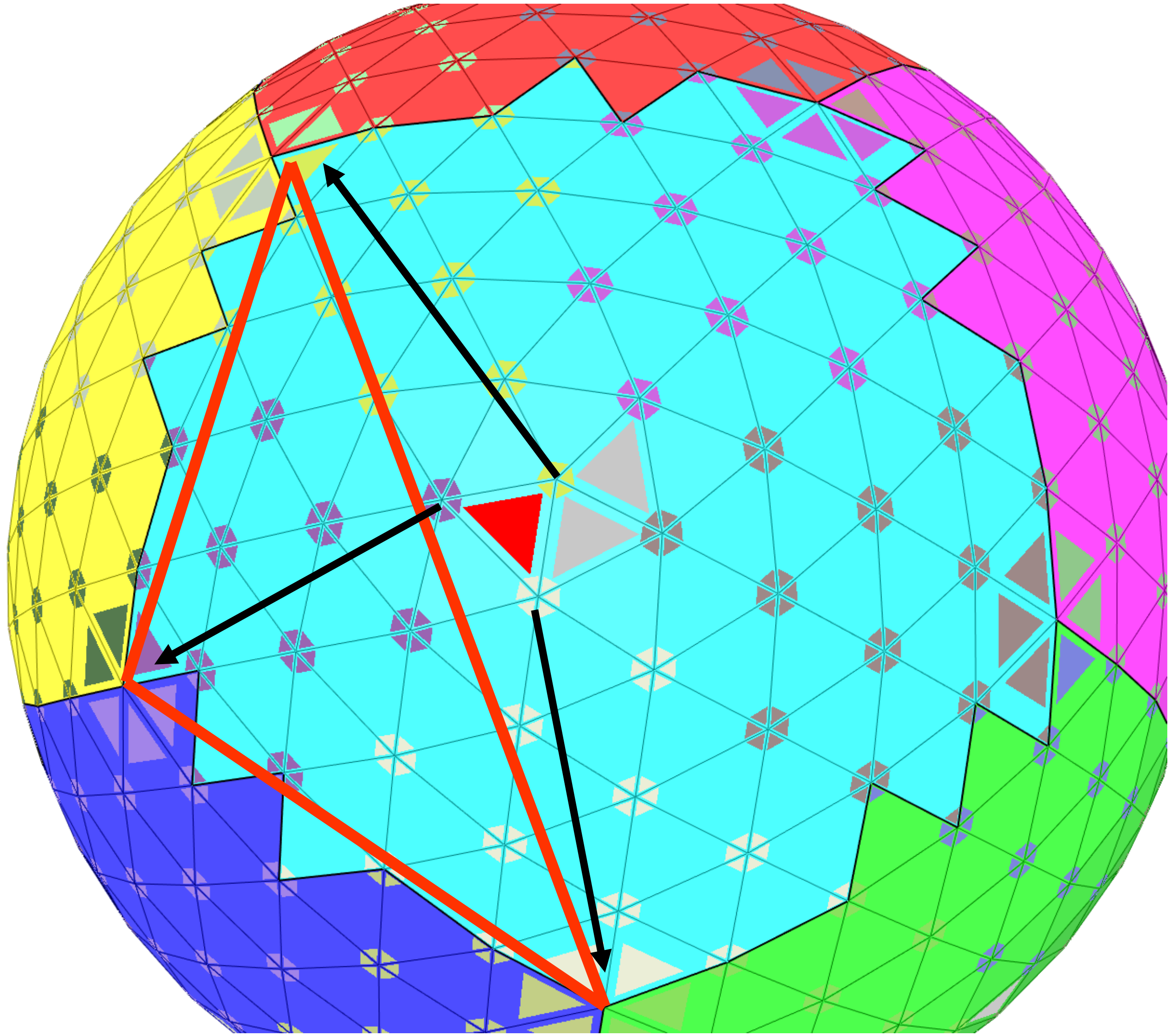


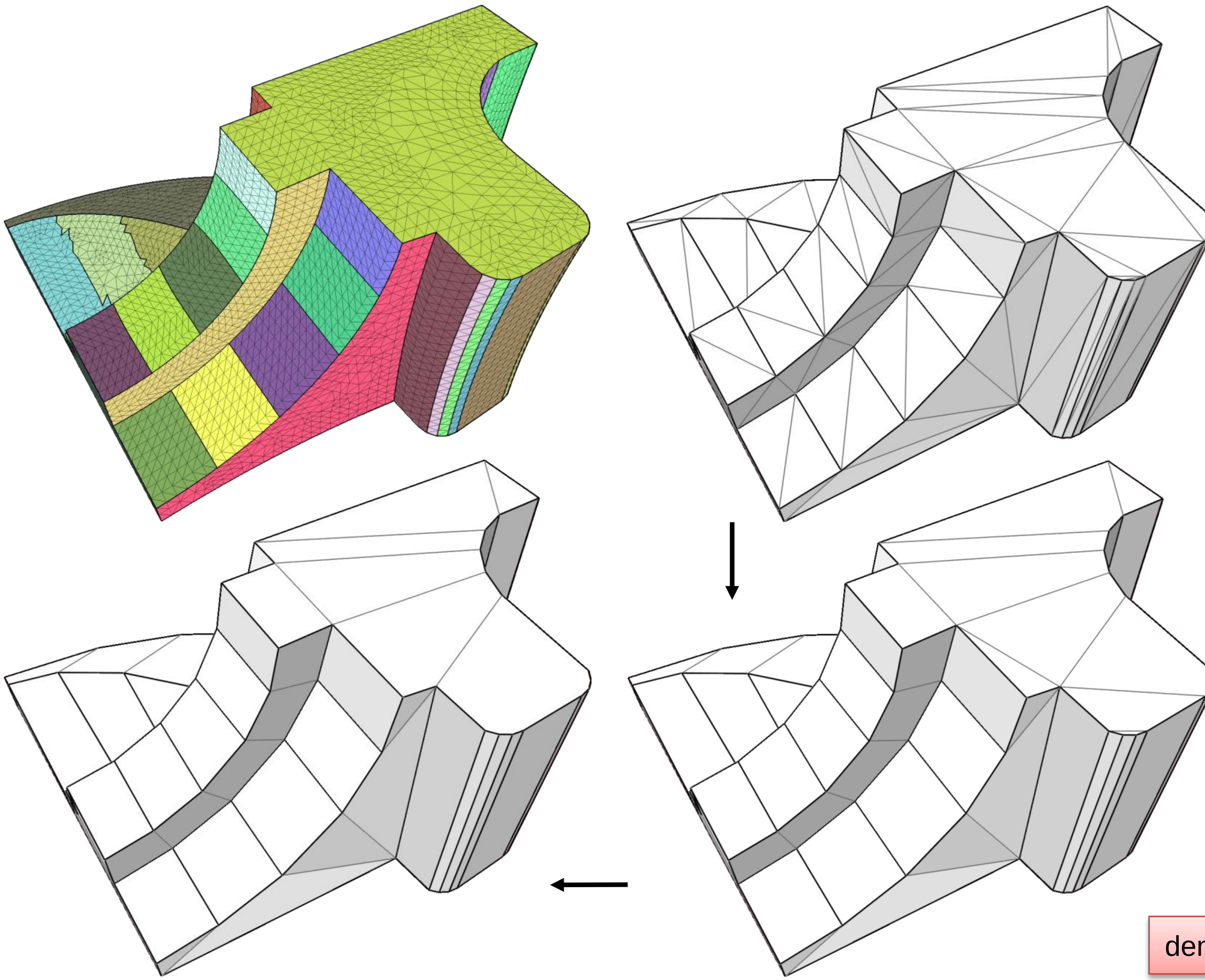
connect 3 source wedges

Triangulation



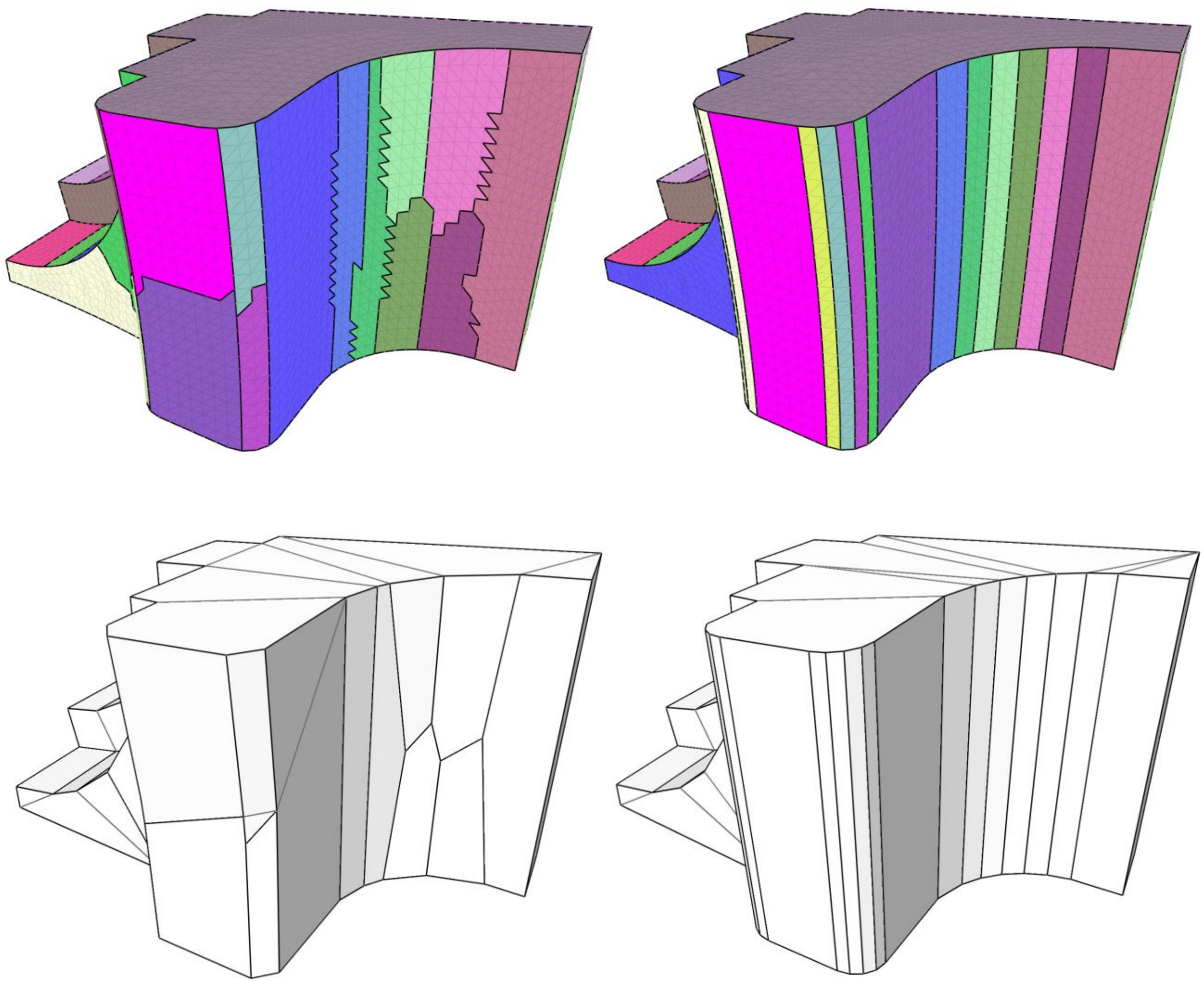






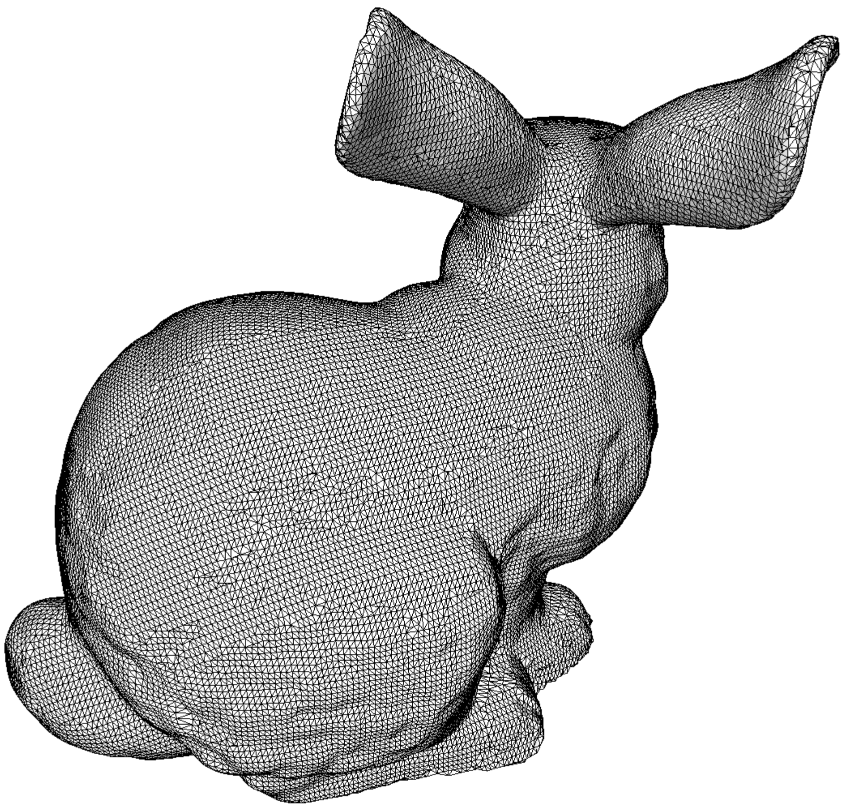
demo

Metrics

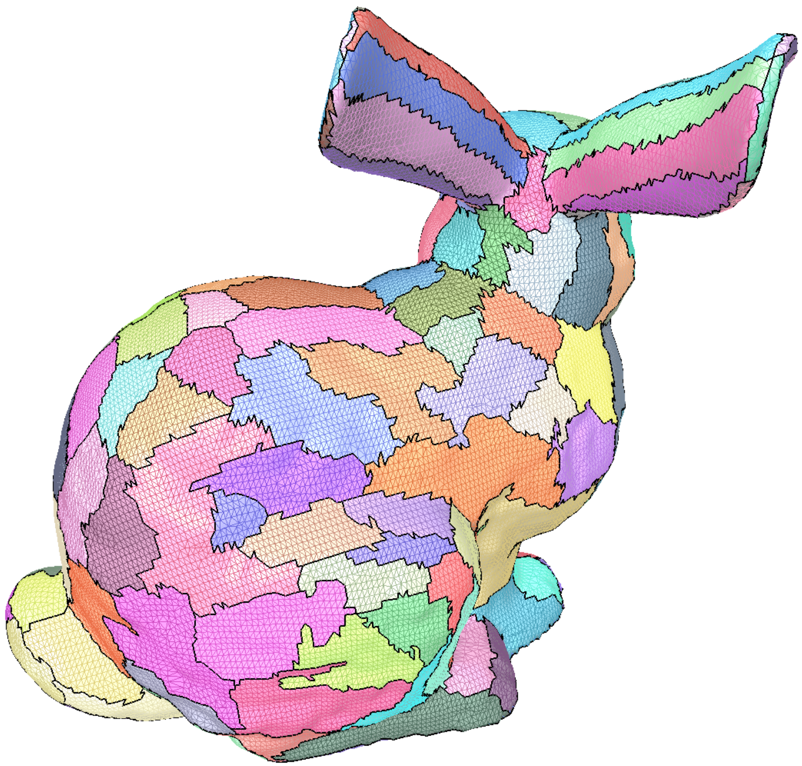


L^2 $L^{2,1}$

Example

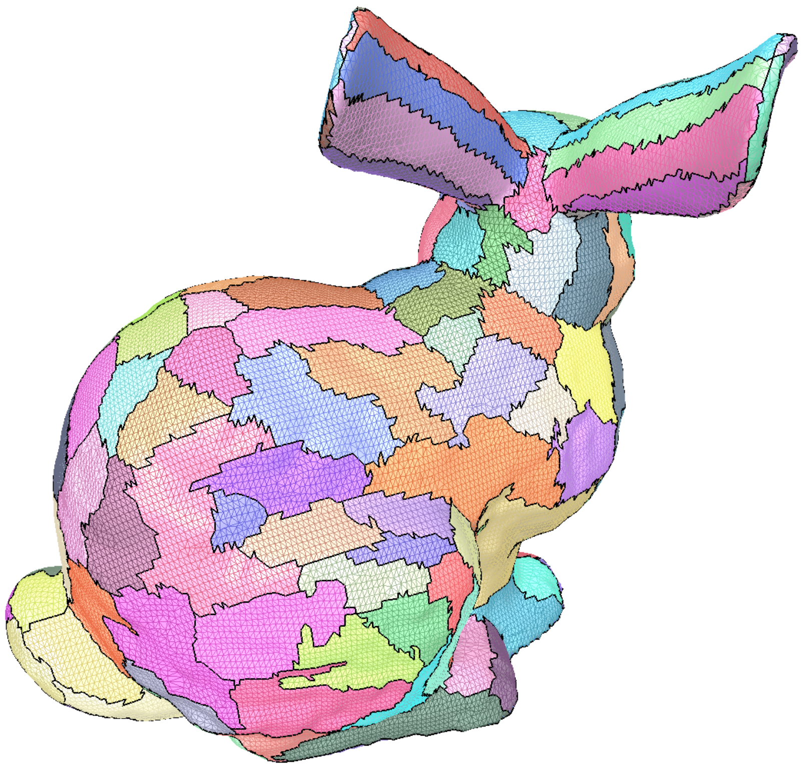


input

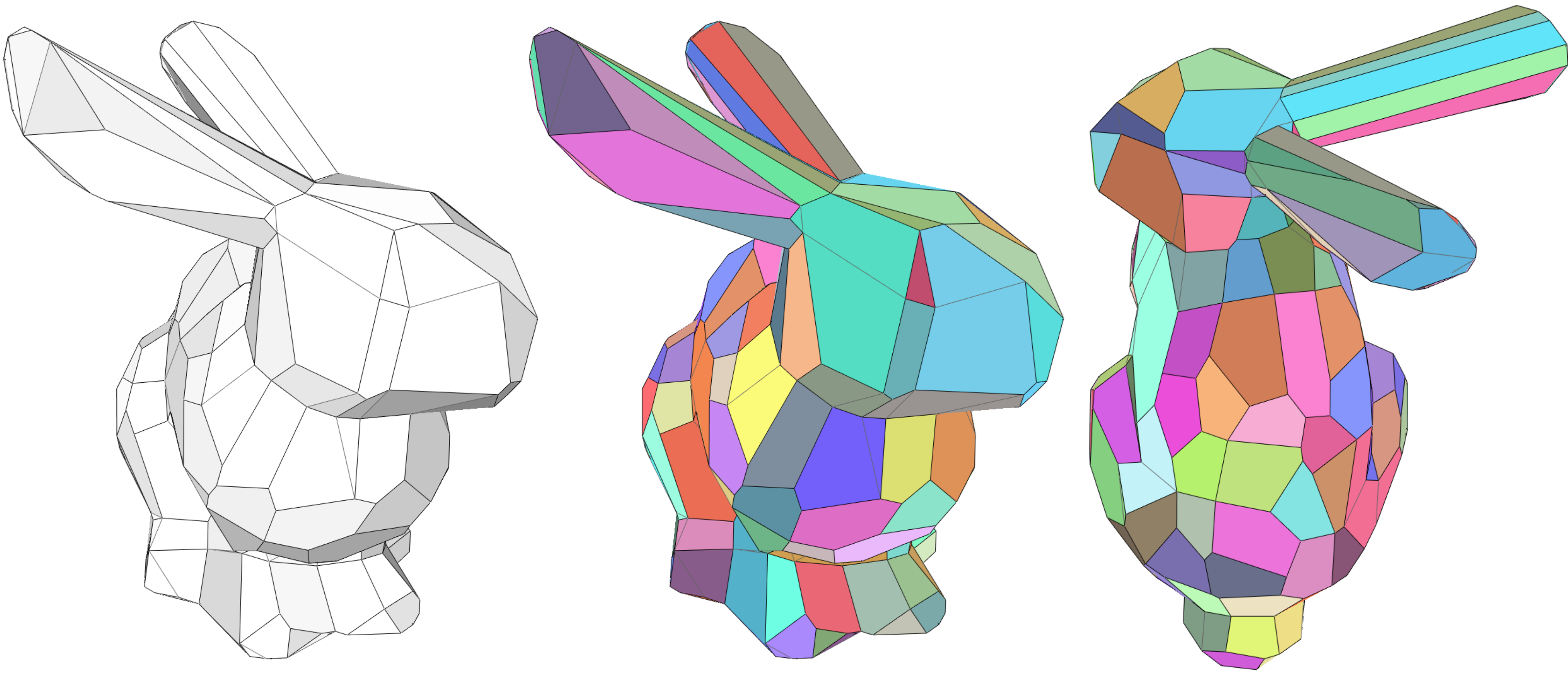


200 proxies

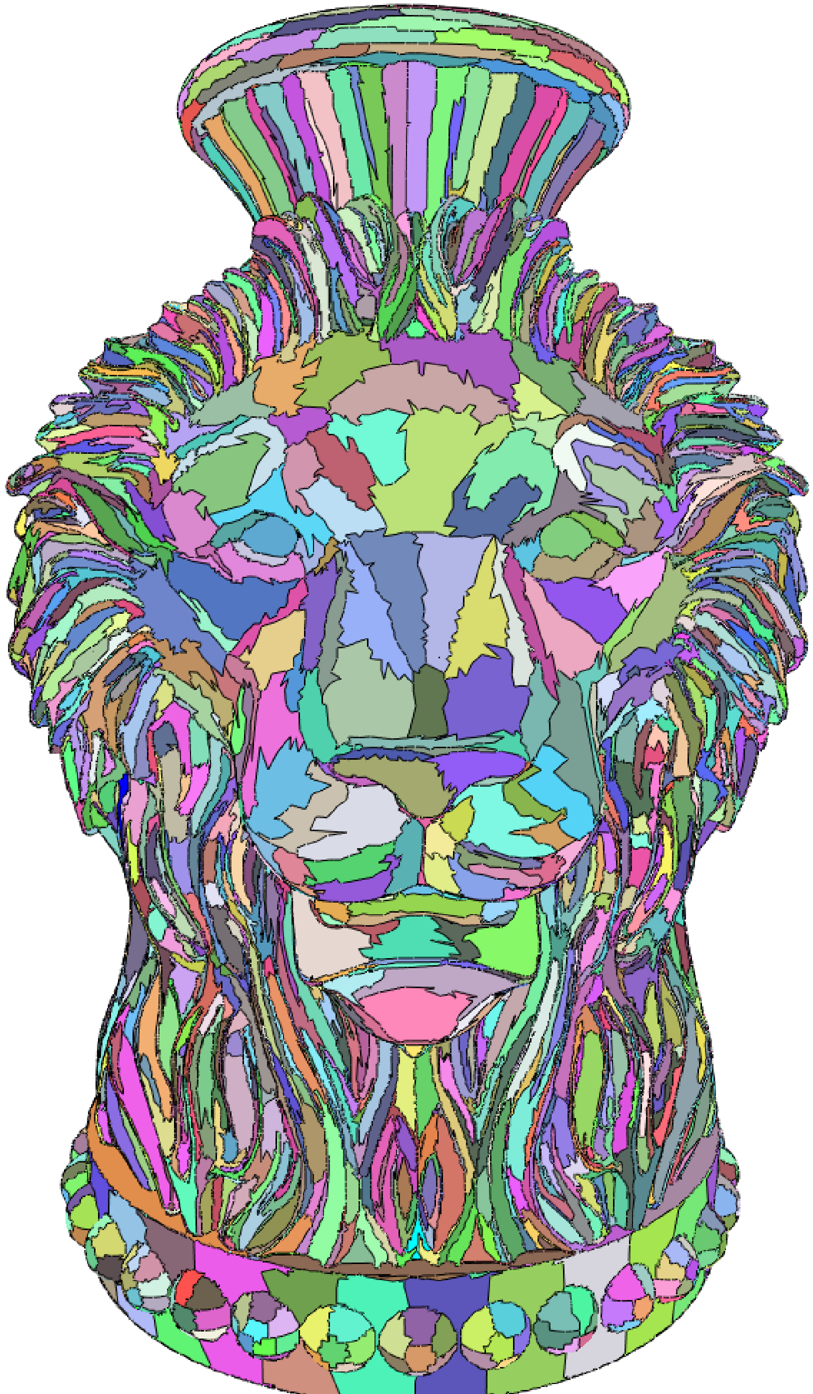
Example

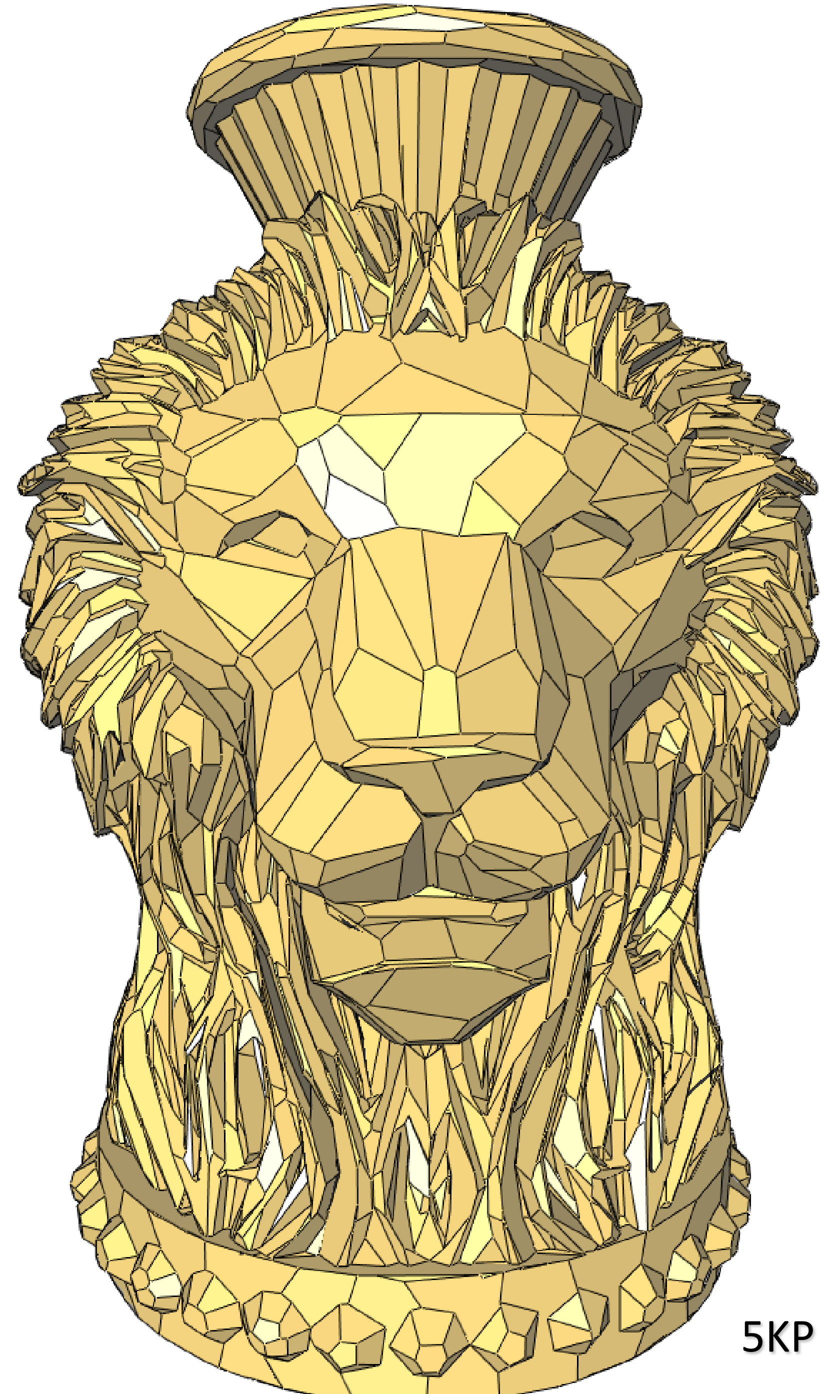


Example

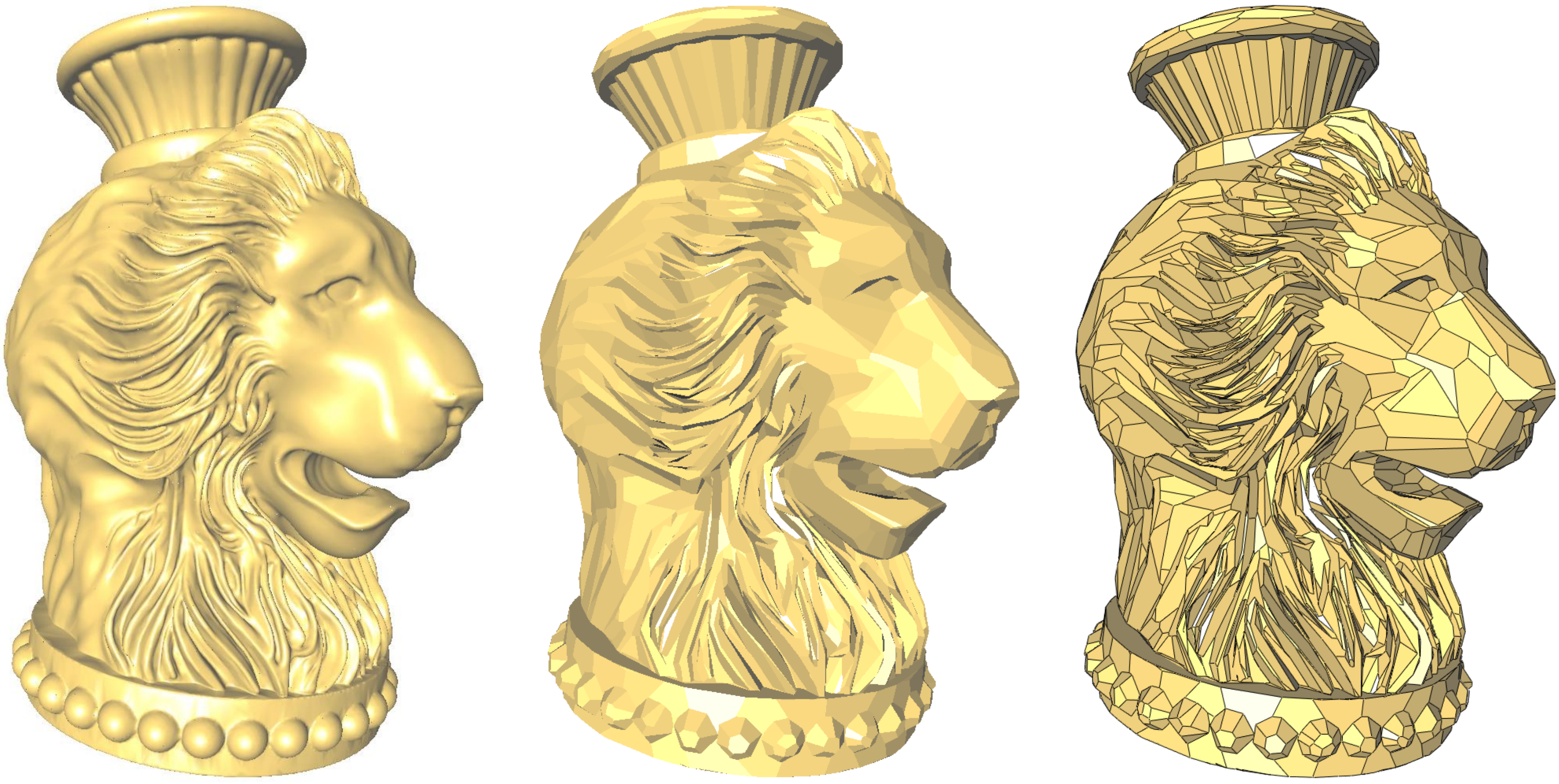



400KT





5KP

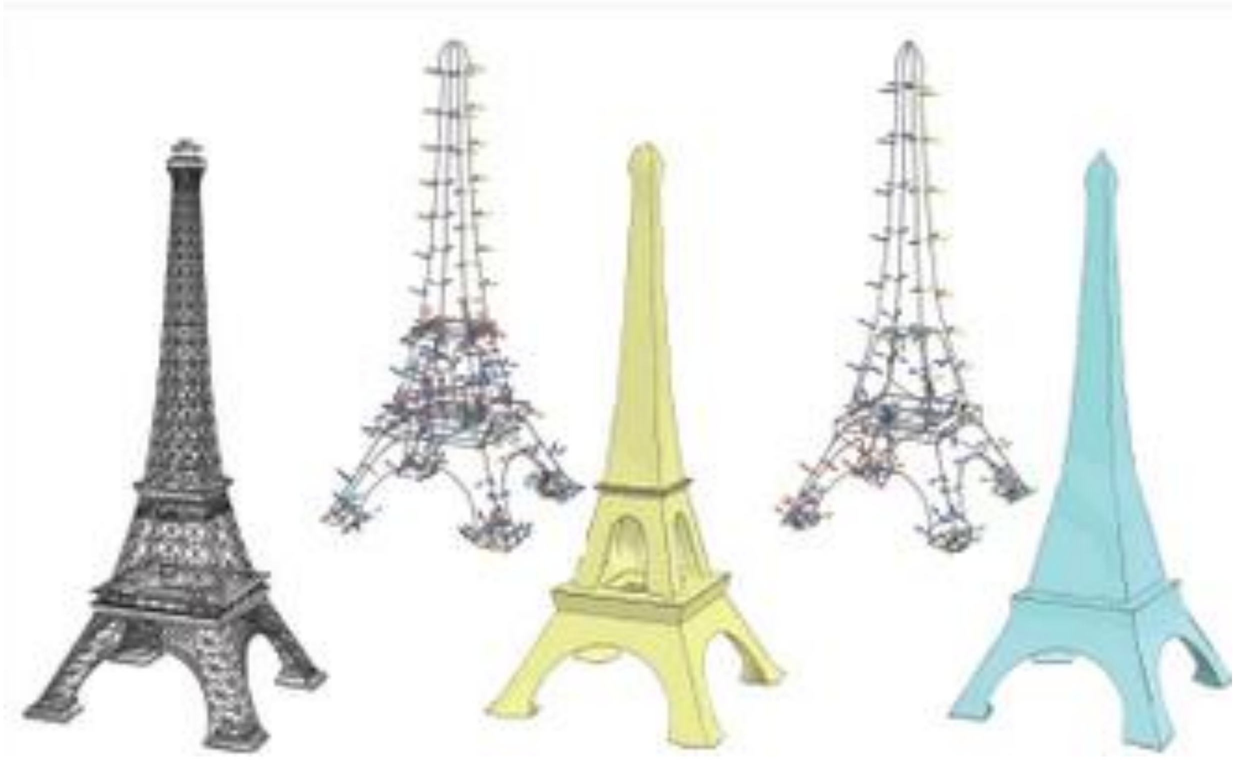


400KT 5KP

Remaining Challenges

Remaining Challenges

- **Beyond approximation**
  - Abstraction [Sheffer, Mitra et al. 2009] *Abstraction of Man-Made Shapes*.



Remaining Challenges

- **Beyond approximation**
  - *Meaningful* LODs. [Verdié, Lafarge, A. 2013]